Fachbereich Mathematik der Universität Hamburg **SuSe 2022**
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Differential Equations II for Engineering Students

Homework sheet 7

Exercise 1: (Vibrating String)

Solve the initial boundary value problem

$$\begin{aligned} u_{tt} &= c^2 u_{xx} && \text{for} \quad 0 < x < 1,\ t > 0, \\ u(0,t) &= u(1,t) = 0 && \text{for} \quad t > 0, \\ u(x,0) &= 0 && \text{for} \quad 0 < x < 1, \\ u_t(x,0) &= \begin{cases} 1, & \frac{1}{20} \le x \le \frac{1}{10}, \\ 0 & \text{else,} \end{cases} \end{aligned}$$

using the suitable product ansatz.
You will get a Fourier series as the solution. Plot the partial sums of the first 20 non-vanishing summands of this series for $c = 2$, $x \in [0,1], t \in [0, 0.4]$ and $t \in [0,2]$.

Exercise 2:

We are looking for an approximation of the solution to the following problem

$$\begin{aligned} & u_{tt} = u_{xx} && x \in (0, 2\pi),\ t > 0, \\ & u(x,0) = \begin{cases} x & 0 < x < \frac{\pi}{2} \\ \pi - x & \frac{\pi}{2} < x < \frac{3\pi}{2} \\ x - 2\pi & \frac{3\pi}{2} < x < 2\pi \end{cases} \\ & u_t(x,0) = 0 && x \in (0, 2\pi) \\ & u(0,t) = u(2\pi, t) = 0 && t > 0 \end{aligned}$$

Sketch the 2π-periodic continuation of the initial data for $x \in [-2\pi, 4\pi]$.

Determine an approximation $\tilde{u}$ to the solution u of the problem using first three terms of the Fourier series.

Check which boundary and initial conditions are already fulfilled by this approximate solution.

Discussion: 11.07.-15.07.2022