Fachbereich Mathematik der Universität Hamburg **SuSe 2022**
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Differential Equations II for Engineering Students

Homework sheet 4

Exercise 1: Given a differential equation

$$2u_{xx} + 4u_{xy} + 2u_{yy} + \sqrt{2}(u_x + u_y) = 0\,.$$

a) Determine the type of the equation.

b) Transform the equation to its normal form.

c) Determine the general solution of the transformed differential equation and perform the backwards transformation.

Exercise 2:

Given the initial value problem

$$\begin{aligned} u_{tt} + u_{xt} - 2u_{xx} &= 0 \quad \text{for } x \in \mathbb{R},\ t \in \mathbb{R}^+ \\ u(x,0) &= \cos(x) \quad \text{for } x \in \mathbb{R}, \\ u_t(x,0) &= -4\sin(x). \quad \text{for } x \in \mathbb{R}. \end{aligned}$$

Solve the problem using the substitution $\alpha = x + t,\ \mu = x - 2t\,.$

Note: The procedure is analogous to the derivation of the solution to the Cauchy problem for the wave equation from the lecture. Alternatively: convert the derivatives in terms of x, t into derivatives in terms of α, μ.

Exercise 3:

a) For which real values of α and for which real-valued functions $g : \mathbb{R} \to \mathbb{R}$ are the following functions harmonic in $\mathbb{R}^2$?

i) $\tilde{u}(x,y) = \cos(\alpha x) \cdot e^{3y}$, ii) $\hat{u}(x,y) = \sin(\alpha x) \cdot \cosh(3y)$

iii) $u(x,y) = \frac{1}{2} \cdot (x^3 + g(x) \cdot y^2)$.

b) Let $\Omega := \left\{(x,y)^T \in \mathbb{R}^2 : x^2 + y^2 < 16\right\}$ and u be the solution of boundary value problem

$$\Delta u(x,y) = 0 \qquad \text{in } \Omega\,, \qquad u(x,y) = \frac{2y^2}{x^2+y^2} \qquad \text{on} \quad \partial\Omega.$$

Determine the value of u in the origin.

Note: $\sin^2(\varphi) = \dfrac{1 - \cos(2\varphi)}{2}$.

Submission deadline: 30.05.-03.06.2022