

# Differential Equations II for Engineering Students

## Work sheet 1

### Exercise 1: (Repetition of DGL I)

- a) Let  $\lambda$  be any fixed real number. Determine a real representation of the general solution to the differential equation

$$y''(t) - \lambda y(t) = 0.$$

- b) Let  $L$  be another fixed positive real number. Determine all solutions to the boundary value problem

$$y''(t) - \lambda y(t) = 0 \quad y(0) = y(L) = 0.$$

For which  $\lambda \in \mathbb{R}$  does the boundary value problem have nontrivial solutions?

The  $\lambda$ -values for which there exist non-trivial solutions (i.e. solutions that are not constantly equal to zero) are called eigenvalues of the problem. The corresponding solutions are called eigenfunctions.

**Remark:** *The solutions to this eigenvalue problem will be needed again and again during the semester!*

### Exercise 2: (Repetition of Analysis II)

Determine the appropriate real Fourier series for the following functions:

- a) Odd  $2L$ -periodic continuation of

$$f : [0, 1[ \rightarrow \mathbb{R}, \quad f(x) = \sin(4\pi x) + 2 \sin(6\pi x) \quad L = 1.$$

- b) Even  $2L$ -periodic continuation of

$$f : [-\frac{\pi}{4}, \frac{5\pi}{4}[ \rightarrow \mathbb{R}, \quad L = \pi \text{ with}$$

$$f(t) = \begin{cases} 2, & -\frac{\pi}{4} \leq t < \frac{\pi}{4}, \\ 0, & \frac{\pi}{4} \leq t < \frac{3\pi}{4}, \\ 2, & \frac{3\pi}{4} \leq t < \frac{5\pi}{4}. \end{cases}$$

**Remark:** *For DGL II you will need to know how to calculate Fourier series. Please repeat if necessary!*

**Discussion: 11-15.04.2022**