Mathematics Department
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SoSe 2022

Exam Differential Equations II

06. September 2022

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I was instructed about the fact that the required test performance will only be assessed if the TUHH examination office can assure my official admission before the exam's beginning.

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Task no.	**Points**	**Evaluater**
1		
2		
3		
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Exercise 1: [7 points]

Given the following initial value problem for $u(x,t)$:

$$u_t + u \cdot u_x = 0, \qquad x \in \mathbb{R},\ t \in \mathbb{R}^+$$

$$u(x,0) = \begin{cases} \frac{1}{2} & x \leq 0, \\ 0 & 0 < x \leq 1, \\ -2 & 1 < x\,. \end{cases}$$

a) Compute the weak solution for $t \in [0, \tilde{t}\,]$ with a sufficiently small $\tilde{t}$.

b) To what maximum t^* can the solution from part a) be continued?

c) Determine the weak solution for $t > t^*$.

Exercise 2: [3 points]

Given is the following differential equation for $u(x,y)$:

$$u_{xx} + 6u_{xy} + (x-y)\,u_{yy} + x^3\,u_x + 5u = 6.$$

Determine the order and the type of the differential equation.

Exercise 3: [7 points]

a) Given the initial boundary value problem

$$\begin{aligned} u_t - 5u_{xx} &= \frac{\pi x}{4}\sin(\pi t) && \text{for } x \in (0,4), t > 0, \\ u(x,0) &= 2\sin(\pi x) + 3\sin(2\pi x) && \text{for } x \in [0,4], \\ u(0,t) = 0, \quad u(4,t) &= 1 - \cos(\pi t) && \text{for } t > 0. \end{aligned}$$

Transform the problem into an initial boundary value problem with homogeneous boundary data using a suitable homogenization of the boundary conditions.

b) Solve the following initial boundary value problem:

$$\begin{aligned} v_t - 5v_{xx} &= 0 && \text{for } x \in (0,4), t > 0, \\ v(x,0) &= 2\sin(\pi x) + 3\sin(2\pi x) && \text{for } x \in [0,4], \\ v(0,t) = 0, \quad v(4,t) &= 0 && \text{for } t > 0. \end{aligned}$$

c) Provide the solution to the initial boundary value problem from part a).

Exercise 4: [1+2 points]

Given the initial value problem

$$\begin{cases} u_{xx} + u_{yy} = 0 & \text{in } \mathbb{R}^2 \\ u = f & \text{on } \mathbb{R} \times \{y = 0\} \\ u_y = g & \text{on } \mathbb{R} \times \{y = 0\} \end{cases}$$

a) When is this initial value problem called well-posed?

b) Using the following ansatz, show that this initial value problem is ill-posed (not well-posed). Choose

$$f \equiv 0, \quad g \equiv 0$$
$$f_n \equiv 0, \quad g_n = \frac{1}{n}\sin(nx)$$

Hint: The solution to the problem with initial values f_n and g_n is

$$u_n(x, y) = \frac{1}{n^2}\sin(nx)\sinh(ny)$$

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