Mathematics Department
Prof. Dr. I. Gasser

WiSe 2021/2022

Mathematik III Exam
(Module: Analysis III)
February 28, 2022

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I was instructed about the fact that the exam performance will only be assessed if the Central Examination Office of TUHH verifies my official admission before the exam's beginning in retrospect.

(Signature)

Exercise	**Points**	**Evaluater**
1		
2		
3		
4		

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Exercise 1: (5 points)

Let

$$f : \mathbb{R}^2 \to \mathbb{R}, \qquad f(x,y) := x^4 - 4xy^3 + 12y + 1.$$

a) Compute the gradient and the Hessian matrix of f.

b) Compute the stationary points of f and classify them.

Exercise 2: (4 points)

The equation

$$f(x,y) := x^2 - x^2 y + \frac{y^3}{3} - 1 = 0$$

is an implicit definition of a curve in $\mathbb{R}^2$.
Show that the implicit function theorem gives us a function g, such that in the neighbourhood of $P_0 := \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ the following equivalence holds

$$f(x,y) = 0 \iff y = g(x), \qquad g(2) = 3\,.$$

Compute the Taylor polynomial of the first degree (the tangent) of g centered at the point $x_0 = 2$.

Exercise 3: (5+2 points)

a) Given are

$$\boldsymbol{D} := \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2 \,:\, 0 \le x^2 + y^2 \le 25,\ x \ge 0,\ y \ge 0 \right\},$$

and a vector field

$$\boldsymbol{f} : \mathbb{R}^2 \to \mathbb{R}^2,\ \boldsymbol{f}(x,y) = \begin{pmatrix} -x^2 y + e^{\tan(x)} \\ xy^2 + \tan(e^y) \end{pmatrix},$$

compute $\mathbf{curl}\,\boldsymbol{f}(x,y)$ and the integral $\displaystyle\int_{\partial D} \boldsymbol{f}(x,y)\, d(x,y)$, where ∂D denotes the positively oriented boundary of $\boldsymbol{D}$.

b) Let f be the vector field

$$\boldsymbol{f} : \mathbb{R}^3 \to \mathbb{R}^3,\ \boldsymbol{f}(x,y,z) = \begin{pmatrix} y^2 + z^2 + 2xz \\ x^2 + z^2 - 2yz \\ x^2 + y^2 - 2xy \end{pmatrix}.$$

Compute div $\boldsymbol{f}(x,y,z)$ and the flux (flow) of $\boldsymbol{f}$ through the surface of the sphere

$$\boldsymbol{K} := \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \,:\, 0 \le (x-1)^2 + (y-2)^2 + (z+3)^2 \le 1 \right\}.$$

Exercise 4: (4 points)

Given the function

$$\boldsymbol{f} : \mathbb{R}^3 \to \mathbb{R}^3, \qquad \boldsymbol{f}(x,y,z) = (-xy\,,\ x^2\,,\ z)^T$$

and the curve

$$\boldsymbol{c} : [0\,,\,2\pi] \to \mathbb{R}^3, \qquad \boldsymbol{c}(t) = (2\cos(t),\ 2\sin(t),\ t)^T.$$

Compute the line integral

$$\int_{\boldsymbol{c}} \boldsymbol{f}(x,y,z)\,d(x,y,z)\,.$$