Mathematics Department
Prof. Dr. I. Gasser

SoSe 2022

Mathematik III Exam
(Modul: Analysis III)

06. September 2022

Please mark each page with your name and your matriculation number.

Please write your surname, first name and matriculation number in block letters each in the designated fields following. These entries will be stored on data carriers.

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I was instructed about the fact that the required test performance will only be assessed if the TUHH examination office can assure my official admission before the exam's beginning.

(Signature)

Task no.	Points	Evaluater
1		
2		
3		

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Exercise 1: (3+1 points)

A local minimum of the function

$$f : \mathbb{R}^2 \to \mathbb{R}\,, \qquad f(x,y) := x^2 + 4y^2 - 6x + 24y + 6\,.$$

subject to the constraint

$$g(x,y) := \cos(\frac{x-3}{2}) + \sin(y+1) - 1 = 0$$

is sought.

$P_0 = (3, -1)^T$ is an admissible point for which the regularity condition is satisfied. This information may be used without proof.

a) Show that P_0 is a stationary point of the corresponding Lagrangian function for a suitable multiplier.

b) Show that $P_0 = (3,-1)^T$ is a local minimum of the function f subject to the constraint $g = 0$ by investigating the sufficient condition of second order.

Exercise 2: (3+3 points)

Consider the function

$$f : \mathbb{R}^2 \to \mathbb{R}, \quad f(x,y) := \; y\cos(x) \;+\; x\sin(y) \;+\; 2.$$

a) Determine the second-degree Taylor polynomial T_2 of f at the point $(x_0, y_0) = (0,0)$.

b) Show that
$$|f(x,y) - T_2(x,y)| \leq \frac{4}{100}$$
for all $(x,y) \in D := [-0.3,\, 0.3] \times [-0.3,\, 0.3]$.

Exercise 3: (5+1+3+1 points)

Consider the half ball $K := \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \, : \, x^2 + y^2 + z^2 \le 4 \, , \; z \le 0 \right\}$

and the vector field $\boldsymbol{f}\,(x, y, z) = \begin{pmatrix} xz + x \\ yz + y \\ x^2 + y^2 \end{pmatrix}$.

a) Compute the integral $\displaystyle\int_K \operatorname{div} \boldsymbol{f}\,(x, y, z)\, d(x, y, z)$.

Hint: $2\sin(\alpha)\cos(\alpha) = \sin(2\alpha)$.

b) The solid K is bounded by a flat surface D and a non-flat surface M . State a parametrization of the flat surface D .

c) Compute the flux (flow) of $\boldsymbol{f}$ through the flat surface D .

d) According to a) and c), what is the flux (flow) of $\boldsymbol{f}$ through the non-flat surface M ?