Fachbereich Mathematik der Universität Hamburg **WiSe 2021/22**
Prof. Dr. I. Gasser
Dr. H. P. Kiani, S. Onyshkevych

Analysis III for Engineering Students

Work sheet 5

Exercise 1:

Given the following optimization problem:

$$\begin{aligned} \text{Find the minima of} \quad & f(x,y) = 2 - x + \frac{4}{9}y \\ \text{that satisfy the constraint} \quad & g(x,y) = 25 - 9x^2 - y^2 \geq 0\,. \end{aligned} \tag{1}$$

a) Are there any local minima in the interior the admissible region, i.e. of $25 - 9x^2 - y^2 > 0$? Explain your answer.

   Hint: local minima in the interior of the admissible set are also the local minima of the unconstrained problem: $\min\limits_{x,y\in\mathbb{R}} \quad f(x,y) = 2 - x + \frac{4}{9}y\,.$

b) Find all global minima of f that satisfy the constraint

$$g(x,y) = 25 - 9x^2 - y^2 = 0$$

   using the Lagrange multiplier rule. First check the regularity condition.

   Remark: This exercise can also be solved by eliminating one of the variables. However, in this exercise we would like to practice the new solution method on the simple example.

c) Find all global minima of the optimization problem (1) .
   Hint: use a) and b).

Exercise 2:

Given the minimization problem:

$$f(x,y,z) := 2x + y + z \to \min$$

subject to

$$g(x,y,z) := x^2 + y^2 + z^2 = 9.$$
$$h(x,y,z) := x^2 + (y - z)^2 = 1.$$

a) Show that $\boldsymbol{x}_0 = (1,2,2)^T$ together with the corresponding multiplier is a stationary point of the Lagrange function $F := f + \lambda_1 g + \lambda_2 h$.

b) Show that the point $\boldsymbol{x}_0 = (1,2,2)^T$ is a local maximum of the function f that fulfills the given constraint. To do this, check second order sufficient condition.

Discussion: 13.12–17.12.21