Fachbereich Mathematik der Universität Hamburg **WiSe 2021/22**
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Analysis III for Engineering Students

Homework sheet 4

Exercise 1 [12 points] Given a function

$$f : \mathbb{R}^2 \to \mathbb{R}, \qquad f(x,y) \; = x \cdot \arctan(y) + e^{x+y} - 1 .$$

a) Compute the second degree Taylor polynomial T_2 of f centered at a point $(0,0)^{\mathrm{T}}$.

b) Show that for the remainder $R_2(x,y) = f(x,y) - T_2(x,y)$ in the area $|x| \le 0.1$, $|y| \le 0.1$ the following estimate holds:

$$|R_2(x,y)| \le 0.006 .$$

c) Find the stationary point of T_2 and check, whether it is minimum, maximum or a saddle point.

Hints: $(\arctan(y))' = \dfrac{1}{1+y^2}$, $\arctan(0) = 0$.

Exercise 2: Given a function $f(x,y) := x^4 + y^4 + 8xy = 0$.

a) (i) Show using the implicit function theorem that $f(x,y)$ can be solved for y near the point $(x_0,y_0)^T := (2,-2)^T$. It means that there exists a function $g(x)$ with $g(2) = -2$, such that in some neighbourhood of x_0 and y_0 the following equivalence holds

$$f(x,y) = 0 \iff y = g(x) .$$

(ii) Compute the first-order Taylor polynomial of function g from the part a) centered at a point $x_0 = 2$.

b) Using the implicit function theorem show that the solution set of

$$f(x,y,z) := (x^2 - 2e^{xy})z + 2 = 0$$

in a neighbourhood of the point $P_0 := (x_0,y_0,z_0)^T := (0,1,1)^T$ can be solved for x. It means that there is a function $g(y,z)$ with $g(1,1) = 0$ such that in a neighbourhood of x_0, y_0, z_0 it holds

$$f(x,y,z) = 0 \iff x = g(y,z) .$$

Using the implicit function theorem for which other variable(s) one can solve the problem?

Submission deadline: 29.11. – 03.12.21