Fachbereich Mathematik der Universität Hamburg **WiSe 2021/22**
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Analysis III for Engineering Students

Work sheet 3

Exercise 1:

Compute the Jacobian matrices for the following functions

a) (i) $\hat{\boldsymbol{f}}_1 : \mathbb{R}^2 \to \mathbb{R}^3, \qquad \hat{f}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x+3y \\ x^2+y^2 \\ xy \end{pmatrix}.$

(ii) $\boldsymbol{f} : \mathbb{R}^2 \to \mathbb{R}^2, \qquad f\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} e^{(3\gamma+1)x_1} - x_2 - 1 \\ 5x_1 + e^{(3\gamma-1)x_2} - 1 \end{pmatrix}, \qquad \gamma \in \mathbb{R}$ fixed parameter.

(iii) $\tilde{\boldsymbol{f}} : \mathbb{R}^3 \to \mathbb{R}^3, \qquad \tilde{f}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -4x_1 + x_2^2 - 3x_3 \\ x_1^2 - 3x_2 + x_3^2 \\ 2x_1 - 4x_2^4 + x_3 \end{pmatrix}.$

b) For the transformation from spherical coordinates to Cartesian coordinates

$$\boldsymbol{g} : \mathbb{R} \times [0, 2\pi] \times [-\tfrac{\pi}{2}, \tfrac{\pi}{2}] \to \mathbb{R}^3, \qquad \boldsymbol{g}\begin{pmatrix} r \\ \phi \\ \theta \end{pmatrix} = \begin{pmatrix} r\cos(\phi)\cos(\theta) \\ r\sin(\phi)\cos(\theta) \\ r\sin(\theta) \end{pmatrix}$$

it holds that

$$\det(\boldsymbol{J}\,\boldsymbol{g}(r,\phi,\theta)) = r^2\cos(\theta).$$

Provide the determinant of the Jacobian matrix for the transformation to elliptic coordinate system

$$\tilde{\boldsymbol{g}} : \mathbb{R} \times [0, 2\pi] \times [-\tfrac{\pi}{2}, \tfrac{\pi}{2}] \to \mathbb{R}^3, \qquad \boldsymbol{g}\begin{pmatrix} r \\ \phi \\ \theta \end{pmatrix} = \begin{pmatrix} ar\cos(\phi)\cos(\theta) \\ br\sin(\phi)\cos(\theta) \\ cr\sin(\theta) \end{pmatrix}$$

for fixed values of a, b, c.

Exercise 2:

Compute the 2-nd order Taylor polynomial of

$$f(x,y,z) = 2 + xz + y^2 + e^x y^2 \cos(z)$$

around a point $(x_0, y_0, z_0)^T := (0, 1, \pi)^T$.

Classes: 15.–19.11.21