Fachbereich Mathematik der Universität Hamburg **WiSe 2021/22**
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Analysis III for Engineering Students

Homework sheet 3

Exercise 1:

Given a function $f:\mathbb{R}^3\to\mathbb{R}$, $f(\boldsymbol{x}) := 2x^2 + y^2 - 4x + z$, a point $\boldsymbol{x}_0 = (1,2,3)^T$, and a direction $\boldsymbol{a} = \frac{1}{\sqrt{6}}(-1,-1,-2)$:

a) Provide the equation of the level surface $N_{\boldsymbol{x}_0}$ of the function f at the point $\boldsymbol{x}_0 = (1,2,3)^{\mathrm{T}}$ and compute the gradient of f at $\boldsymbol{x}_0$.

b) Compute the directional derivative $D_{\boldsymbol{a}} f(\boldsymbol{x}_0)$ in the direction $\boldsymbol{a} = \frac{1}{\sqrt{6}}(-1,-1,-2)^{\mathrm{T}}$.

Can you determine whether it is a direction of ascent or descent? Can you tell whether the function values increase or decrease when one moves from $\boldsymbol{x}_0$ in the direction $\boldsymbol{a}$?

c) Compute the function values $f(\boldsymbol{x}_0 + t\,\boldsymbol{a})$ for $t = \frac{\sqrt{6}}{2}, 2\sqrt{6}, 3\sqrt{6}$.
Is there a contradiction to your result from b?

Exercise 2:

Let $\boldsymbol{u} = (u(x,y), v(x,y))^{\mathrm{T}}$ be a velocity field of the two-dimensional flow, $r = \sqrt{x^2+y^2}$ and $\epsilon \in \mathbb{R}^+$. Given the velocity fields

a) $u = \epsilon x, \quad v = \epsilon y$

b) $u = \epsilon \dfrac{x}{r^2}, \quad v = \epsilon \dfrac{y}{r^2}, \ (x,y) \neq (0,0)$ (isolated source)

c) $u = \epsilon \dfrac{-y}{r^2}, \quad v = \epsilon \dfrac{x}{r^2}, \ (x,y) \neq (0,0)$ (isolated vortex)

compute the source density $\operatorname{div} \boldsymbol{u}$ and vortex density $\operatorname{rot} \boldsymbol{u} := v_x - u_y$. Sketch the vector fields and a few associated streamlines (they are the solutions of the system of differential equations $\dot{x} = u,\ \dot{y} = v$ or the differential equation $y'(x) = v(x,y)/u(x,y)$).

Submission deadline: 15.–19.11.21