Fachbereich Mathematik der Universität Hamburg **WiSe 2021/22**
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Analysis III for Engineering Students

Homework sheet 2

Problem 1:

a) Find all first and second order partial derivatives of

$$s(x,y,z) := xyz\,\sin(x+y+z) \qquad \text{and} \qquad g(x,y,z) := \frac{\cos^2(x)e^y}{z}\,.$$

b) Calculate for the function $f:\,\mathbb{R}^3 \longrightarrow \mathbb{R}$

$$f(x,y,z) = \arctan(x)e^y + \sin(x)\ln(1+y^2)z + x^2e^{z^2}$$

the derivative f_{xyz} as well as $\nabla f(x,y,z)$.

Problem 2: The function

$$u(x,t) := \frac{1}{2}\left[\sin\left(\frac{2\pi}{L}(x+ct)\right) + \sin\left(\frac{2\pi}{L}(x-ct)\right)\right]$$

describes approximately the displacement of the point $x \in [0,L]$ of a vibrating string of length L at time $t > 0$

The position of the string at time $t = 0$ is $u(x,0) = \sin\left(\frac{2\pi x}{L}\right)$. These are the so-called initial values.

a) Calculate the displacement at the end points of the string, the so-called boundary values $u(0,t)$ and $u(L,t)$.

b) Show that u satisfies the wave equation $\dfrac{\partial^2 u}{\partial t^2} = c^2 \dfrac{\partial^2 u}{\partial x^2}$.

c) Try to sketch the form of the string for $t = 0, \frac{L}{6c}, \frac{L}{4c}, \frac{L}{3c}, \frac{L}{2c}, \frac{L}{c}$.
Hint: $\sin(a+b) + \sin(a-b) = 2\sin(a)\cos(b)$.

Due date: 01.–05.11.21