

Analysis III for Engineering Students Work sheet 1

Exercise 1: Consider the following sets

$$M_1 := \left\{ \begin{pmatrix} x \\ y \end{pmatrix} : x, y \in \mathbb{R}, x^2 + y^2 \leq 1 \right\},$$

$$M_2 := \left\{ \begin{pmatrix} x \\ y \end{pmatrix} : x, y \in \mathbb{R}, x^2 + y^2 < 4 \right\},$$

$$M_3 := \left\{ \begin{pmatrix} x \\ y \end{pmatrix} : x, y \in \mathbb{R}, 1 \leq x^2 + y^2 < 4 \right\},$$

$$M_4 := \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : x, y, z \in \mathbb{R}, x^2 + y^2 \leq 1 \right\},$$

$$M_5 := \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : x, y, z \in \mathbb{R}, x^2 + y^2 + z^2 < 1 \right\},$$

$$M_6 := \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2 : (x, y) \cdot (1, 2)^T = 1 \right\},$$

$$M_7 := \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 : (x, y, z) \cdot (1, 2, 1)^T < 1 \right\},$$

$$M_8 := \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : x, y, z \in \mathbb{R}, z = x^2 + y^2 \right\}.$$

$$M_9 := \left\{ \begin{pmatrix} x \\ y \end{pmatrix} : x, y \in \mathbb{R}, (x + 3)^2 + y^2 \leq 1 \right\} \cup \left\{ \begin{pmatrix} x \\ y \end{pmatrix} : x, y \in \mathbb{R}, (x - 3)^2 + y^2 \leq 1 \right\}.$$

- a) Which are the boundary points of M_1, \dots, M_9 ?
- b) Decide for each set M_1, \dots, M_9 if it is closed, open or neither closed nor open.
- c) Which of the sets M_1, \dots, M_9 are bounded?
- d) Which sets M_1, \dots, M_9 are connected? Which are convex?

Exercise 2: Consider the functions $f_k : \mathbb{R}^2 \rightarrow \mathbb{R}$, $k = 1, 2, 3, 4$

a) $f_1(x, y) = 2x + 3y,$

b) $f_2(x, y) = x^2 + \frac{y^2}{9},$

c) $f_3(x, y) = \cos(x - y^2),$

d) $f_4(x, y) = \exp(x \cdot y).$

Draw a few contour lines (curves along which the function has a constant value)

$$f_k^{-1}(C) := \{(x, y)^T : f(x, y) = C\}$$

for each f_k .

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