041207 (1)

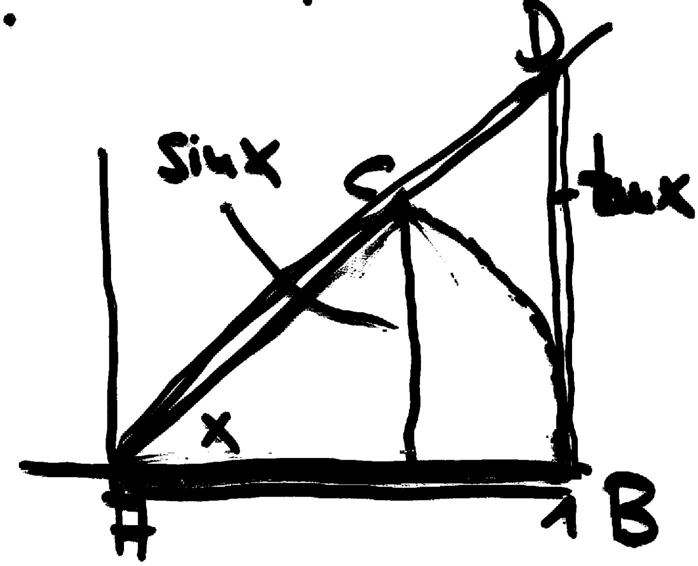
$$\lim_{x \to 0+0} f(x) = 1 \underset{f(0)}{=}$$ rechtsseitiger Grenzwert existiert

$$\lim_{x \to 0-0} f(x) = 0 \underset{f(0)=1}{\neq}$$ linksseitiger Grenzwert existiert

<u>Weiteres</u> <u>Bsp</u> : $f(x) = \frac{\sin x}{x}$

$D_f = \mathbb{R} \setminus \{0\}$

$$\lim_{x \to 0} f(x) = ? = 1$$



$\Delta(ABC) \leq S(ABC)$

$\leq \Delta(ABD)$

d.h.

$$\frac{1}{2}\sin x \leq \frac{x}{2\pi}\pi \leq \frac{1}{2}\tan x \quad : \sin x$$

$$1 \leq \frac{x}{\sin x} \leq \frac{1}{\cos x}$$

041207 ②

$$\lim_{x\to 0} \frac{1}{\cos x} = 1$$

$$\Rightarrow \quad \lim \frac{x}{\sin x} = 1 = \lim \frac{\sin x}{x} \quad ✓$$

Sinnvolle ~~Erweiterung~~ Fortsetzung von $\frac{\sin x}{x}$:

$$f(x) = \begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$$

Dann $\lim_{x\to 0} f(x) = 1 = f(0)$

δ- Kugel $U_\delta(x_0)$ offen

$$U_\delta(x_0) := \{x \in \mathbb{R};\ |x - x_0| < \delta\}$$

Bsp Stetigkeit / Unstetigkeit

$$f(x) := \begin{cases} |x|, & x \neq 0 \\ 2, & x = 0 \end{cases}$$

$$\lim_{x\to 0} f(x) = 0 \neq f(0) = 2$$

Unstetigkeit "hebbar"

$$\tilde f(x) := |x|, \quad x \in \mathbb{R}$$

Dann gilt $\tilde f(x) = f(x) \quad \forall_{x \neq 0}$

und $\lim_{x\to 0} \tilde f(x) = 0 = \tilde f(0)$,

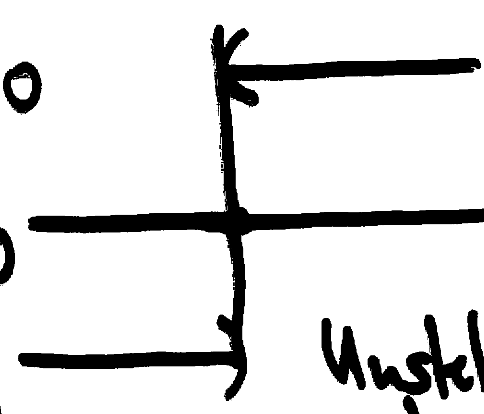
d.h. $\tilde f$ stetig

i) $\cancel{f(x) = \frac{x}{|x|}, \; x \neq 0}$

$$f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

$$\lim_{x\to 0+0} f(x) = 1 \neq f(0) = 0$$

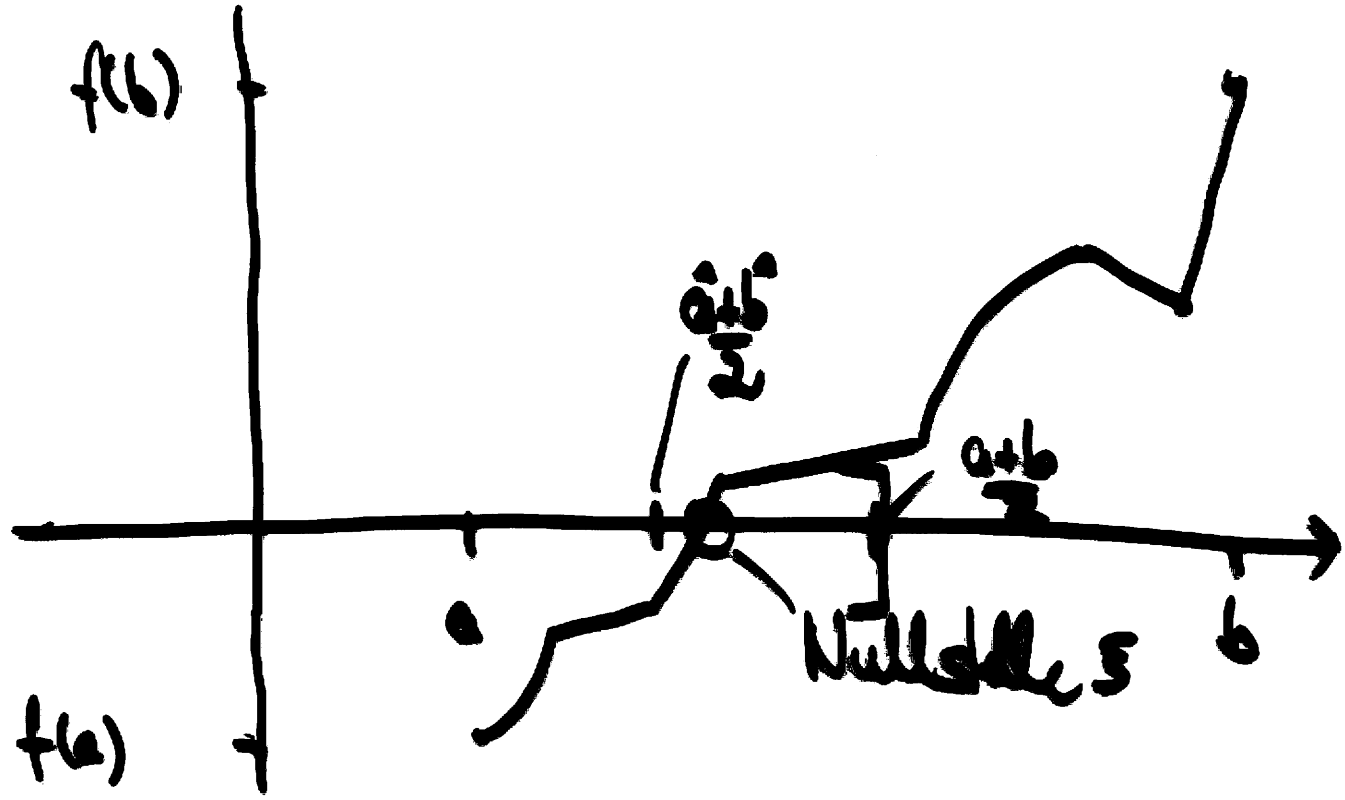
$$\lim_{x\to 0-0} f(x) = -1 \neq f(0) = 0$$



Unstetigkeit nicht hebbar → Unstetigkeit erster Art

Skizze Nullstellensatz

$f(a) \cdot f(b) < 0$



i.) $f(\frac{a+b}{2}) > 0$ $\qquad b^1 := \frac{a+b}{2}, \; a^1 := a$

ii.) $f(\frac{a^1+b^1}{2}) < 0$ $\qquad b^2 := b^1, \; a^2 := \frac{a^1+b^1}{2}$

$\vdots \qquad \vdots$

liefert (a^i), (b^i)

mit (a^i) monoton wachsend

(b^i) " fallend

$a^i - b^i = -2^{-(i+1)}(b-a) \longrightarrow 0 \quad i \to \infty$

$\longrightarrow \lim a^i = \lim b^i = \xi$

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f stetig $\Rightarrow$

$$0 \geq \lim_{i \to \infty} f(a^i) = f(\xi)$$

$$0 \leq \lim_{i \to \infty} f(b^i) = f(\xi)$$

$\Big\} \Rightarrow f(\xi) = 0$

<u>Bsp</u>: $p(x) = -\frac{x^3}{3} + x - \frac{1}{2}$ $I = [0,1]$

$a = 0$, $b = 1$ $p(0) = -\frac{1}{2}$, $p(1) = \frac{1}{6}$

Algorithmus zur Findung von ξ: Bisektion

$i = 0$, $a_0 = 0$, $b_0 = 1$, $\frac{a_0 + b_0}{2} = \frac{1}{2}$, $p(\frac{1}{2}) = -\frac{1}{24}$

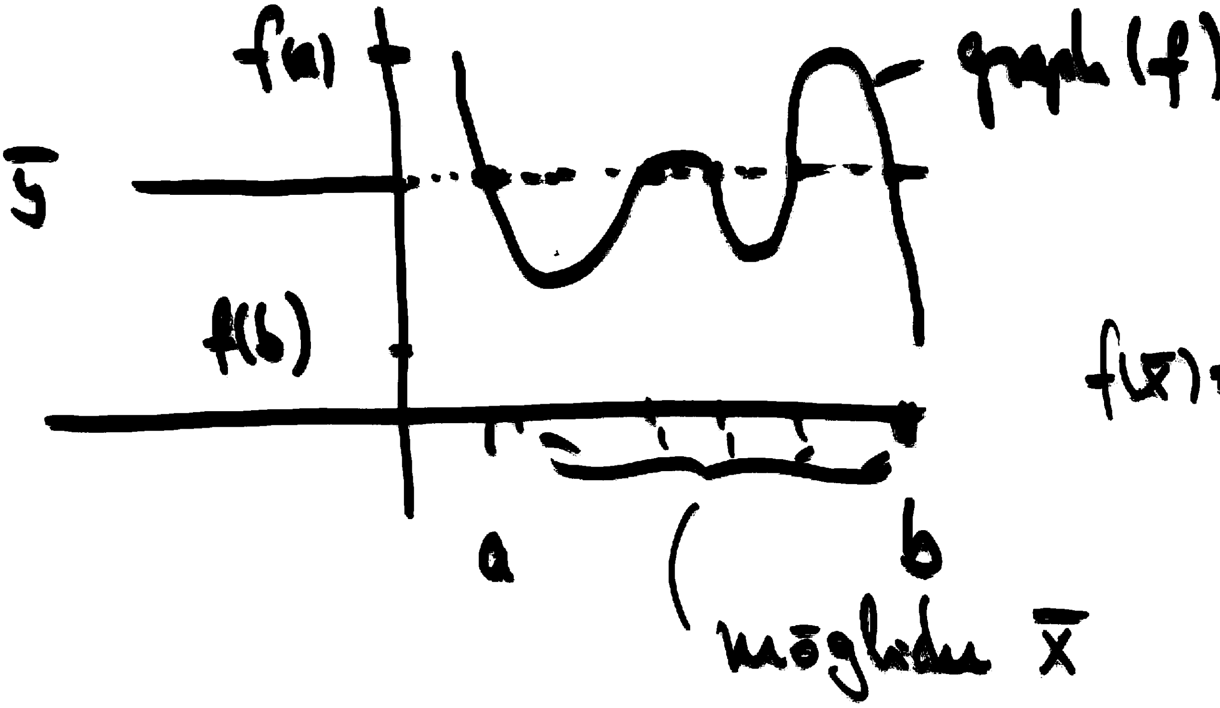
$i = 1$, $a_1 = \frac{1}{2}$, $b_1 = 1$, $\frac{a_1 + b_1}{2} = \frac{3}{4}$, $p(\frac{3}{4}) = \frac{7}{64}$

$i = 2$, $a_2 = \frac{1}{2}$, $b_2 = \frac{3}{4}$, $\frac{a_2 + b_2}{2} = \frac{5}{8}$, $p(\frac{5}{8}) < 0$

$i = 3$, $a_3 = \frac{5}{8}$, $b_3 = \frac{3}{4}$, $\frac{a_3 + b_3}{2} = \frac{11}{16}$, $p(\frac{11}{16}) \ldots$

erzeugt Folge $\frac{1}{2}, \frac{3}{4}, \frac{5}{8}, \frac{11}{16}$. strebt gegen ξ mit $p(\xi) = 0$.

Zwischenwertsatz



$f(\bar{x}) = \bar{y}$

Zwischenwertsatz

$\hat{=}$ Nullstellensatz

für $g(x) = f(x) - \bar{y}$

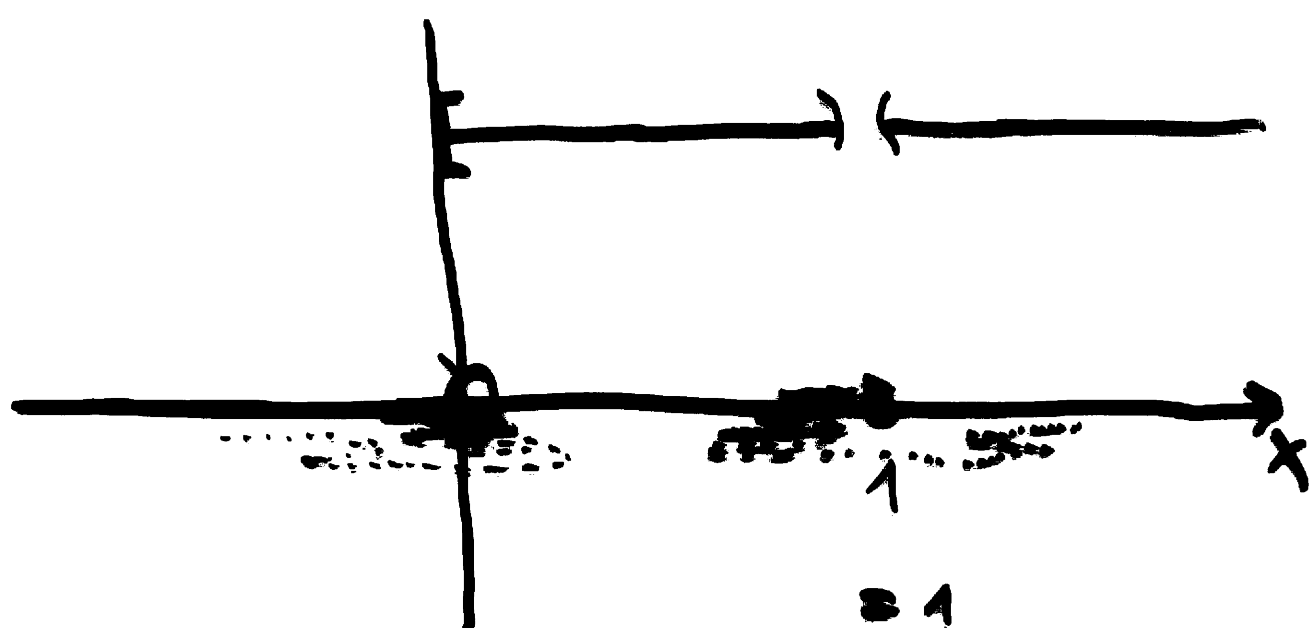
$g(\bar{x}) = 0$ gdw $f(\bar{x}) - \bar{y} = 0$

gdw $f(\bar{x}) = \bar{y}$

27.11.07 (7)

Grenzwerte von Funktionen

Bsp: $$f(x) := \begin{cases} 0, & x<0 \text{ oder } x=1 \\ 1, & \text{sonst} \end{cases}$$



$$\lim_{x\to 1} f(x) = \lim_{x_n\to 1} \overbrace{f(x_n)}^{=1} = 1 = g$$

$$\lim_{n\to\infty} x_n = 1 \qquad \neq f(1) = 0$$

$$g \neq f(1)$$

obwohl $\lim x_n = 1$

$\lim_{x\to 0} f(x) = ?$ existiert nicht