UNIVERSITEIT VAN AMSTERDAM
INSTITUTE FOR LOGIC, LANGUAGE AND COMPUTATION

Core Logic

2007/2008; 1st Semester
dr Benedikt Löwe

Homework Set # 6 *Deadline:* October 24th, 2007

Exercise 19 (3 points).

Consider the following $n+1$-player game: There are n players called **agents**, denoted by a_i, and one distinguished player denoted by G. The game has two rounds; in the first round, the agents simultaneously and independently make a decision α_i between two actions, good and bad; in the second round, after seeing the decisions of the agents, G makes n decisions (one for each player) $\gamma_i \in \{\oplus, \ominus\}$. A **run** of the game is a function $F : i \mapsto \langle \alpha_i, \circ \rangle$ where $\circ \in \{\oplus, \ominus\}$. If F is a run of the game, we say that $\circ$ **is the fate of agent** a_i **in** F if $F(i) = \langle \alpha_i, \circ \rangle$.

Use this game model to analyse the following two texts from the *Catholic Encyclopedia*:

> **The Catholic Encyclopedia on Predestination.** The principal question then is: Does the natural merit of man exert perhaps some influence on the Divine election to grace and glory? If we recall the dogma of the absolute gratuity of Christian grace, our answer must be outright negative.
>
> **The Catholic Encyclopedia on Grace.** Beside the necessity of actual grace, its absolute gratuity stands out as the second fundamental question in the Christian doctrine on this subject. The very name of grace excludes the notion of merit. But the gratuity of specifically Christian grace is so great and of such a superior character that even mere natural petition for grace or positive natural dispositions cannot determine God to the bestowal of his supernatural assistance.

Define formally what *strategies* for the players in the game would be (1 point) and find a property for G's strategy that corresponds to the "absolute gratuity of Grace" (1 point). If τ is a strategy for G, we say that "**the fate of** a_i **is predetermined relative to** τ if there is some $\circ \in \{\oplus, \ominus\}$ such that for every run of the game in which G plays according to τ, the fate of a_i is $\circ$.
Prove that if τ satisfies "absolute gratuity of Grace", then the fate of every agent is predetermined relative to τ (1 point).

Exercise 20 (9 points).

Consider a nonempty set W of states and a nonempty set X of objects. We call the set $\widehat{X} := \{+,-\} \times X := \{\langle +, x\rangle\, ;\, x \in X\} \cup \{\langle -, x\rangle\, ;\, x \in X\}$ the set of **entities**. We think of $\langle -, x\rangle$ as the imagined object x and $\langle +, x\rangle$ as "$\langle -, x\rangle$ with the added property of existence". We call entities $\langle +, x\rangle$ **existing entities**.
For each $w \in W$, fix a nonempty set $X_w \subseteq \widehat{X}$ of **permissible entities** in w. We fix two strict linear orderings $<$ and $\prec$ on $\widehat{X}$, and an accessibility relation R on W. We say that "v is conceivable from w" if wRv. For $x, y \in X_w$, we say "in w, x is better (bigger) than y" if $y < x$ ($y \prec x$). A structure $\mathbf{W} := \langle W, \langle X_w\, ;\, w \in W\rangle, R, <, \prec\rangle$ is called **Anselmian** if it has the following properties:

- If wRv and $\widehat{x} \in X_w$, then $\widehat{x} \in X_v$.

- For each $w \in W$, if $\langle -, x\rangle \in X_w$, then there is some v such that wRv and $\langle +, x\rangle \in X_v$.
- For each $x \in X$, $\langle -, x\rangle < \langle +, x\rangle$.

If **W** is an Anselmian structure and $w \in W$, we say that an entity $\widehat{x} \in \widehat{X}$ is **Anselmian** in w if for all v such that wRv and all $\widehat{y} \in X_v$, it is not the case that $\widehat{x} < \widehat{y}$. We say that an entity $\widehat{x} \in \widehat{X}$ is **Gaunilan** in w if for all v such that wRv and all $\widehat{y} \in X_v$, it is not the case that $\widehat{x} \prec \widehat{y}$.

Give an example of an Anselmian structure in which there is a state w without an Anselmian entity in w (2 points).

The second half of the ontological argument can now be rephrased as follows: In an Anselmian structure, every Anselmian entity is existing. Prove this statement (2 points).

Give an example of an Anselmian structure with a state w in which there is a nonexisting Gaunilan entity (*i.e.*, an entity of the form $\langle -, x\rangle$). (3 points)

There is a simple modification of the notion of an Anselmian structure that we could call a **Gaunilan structure**, for which we can prove that every Gaunilan entity is existing. Give a precise definition of this and prove the statement. (2 points)

Exercise 21 (7 points).

Read the text

> Paul Vincent **Spade**, *Why Don't Mediaeval Logicians Ever Tell Us What They're Doing? Or, What Is This, A Conspiracy?, preprint* 2000

(PDF file on the course webpage) and answer the following questions:

(1) What are Spade's four 'exhibits' for the thesis that "*we simply don't know what is going on*"? (¼ point each)
(2) According to Spade, what does Richard Billingham mean by "immediate terms"? (2 point)
(3) Spade is not concerned that Billingham's proof of "A man runs" doesn't prove anything we didn't know before. What is it that causes Spade trouble with Billingham's example? (2 points)
(4) Would Spade subscribe to the following statements (1 point each):
   (a) 'We don't understand medieval logic because we don't have a full grasp of the underlying medieval philosophy.'
   (b) 'For the theories mentioned in the four exhibits, the historically earliest texts are lost, and this is the main reason why we don't understand what is going on.'

Exercise 22 (4 points).

Many medieval authors think of disjunction as an operator on finite sets of sentences and define $\mathsf{MD}(A_1, ..., A_n)$ to be true if exactly one of the A_i is true.
If f is a binary truth function (i.e., a function from $\{0,1\} \times \{0,1\}$ to $\{0,1\}$), we can use it to recursively define n-ary truth functions by

$$\begin{aligned} f_2(A, B) &:= f(A, B) \\ f_{n+1}(A_0, ..., A_n) &:= f(f_n(A_0, ..., A_{n-1}), A_n). \end{aligned}$$

We say that an n-ary truth function g is **induced by** f if $g = f_n$. Show that medieval disjunction MD is not induced by any binary truth function.