UNIVERSITEIT VAN AMSTERDAM
INSTITUTE FOR LOGIC, LANGUAGE AND COMPUTATION

Core Logic

2007/2008; 1st Semester
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Homework Set # 10 *Deadline:* November 21st, 2007

Exercise 34 (7 points).

It is often said that Frege was an anti-semite. What is our source for this claim? Please describe the source with full bibliographical data (2½ points).

Frege's anti-semitism is often contrasted with his work in logic. He is sometimes mentioned as an example for a researcher with unacceptable political views that had no influence on his research. Find a scholarly source for a claim like this (2½ points).

A famous Frege expert X was shocked when he find out that Frege held anti-semitic opinions. A reviewer Y of X's major work on Frege writes "This was a great shock for X but not for me". Y had gathered from Frege's choice of example sentences in his logical texts that he was very much in line with late XIXth century German conservative thought. Who are X and Y? (1 point each; give bibliographic references to Y's review and the 'major work' of X)

Important. In this exercise, all citations have to be sources that could also be cited in a historical research paper, i.e., published books or journal articles in respectable scientific journals. Quoting `wikipedia.org` or some webpage is not acceptable and will give no credit.)

Exercise 35 (7 points).

Let $\mathcal{L} := \{+, \cdot, 0, 1, -\}$ be the language of Boolean algebras and Φ_{BA} be the axioms of Boolean algebras. Let

$$\varphi \;:\hat{=}\; \forall x \forall y \Big(\big((x \neq x \cdot y) \wedge (y \neq x \cdot y) \big) \rightarrow (x \cdot y = 0) \Big),$$

$$\psi \;:\hat{=}\; \exists x \big((x \neq 0) \wedge (x \neq 1) \big).$$

Let Φ_0, Φ_1, Φ_2, and Φ_3 be the deductive closures of Φ_{BA}, $\Phi_{\mathrm{BA}} \cup \{\neg\psi\}$, $\Phi_{\mathrm{BA}} \cup \{\varphi\}$, and $\Phi_{\mathrm{BA}} \cup \{\varphi, \psi\}$, respectively. Investigate whether Φ_i is a complete theory. If it isn't, give a formula σ such that $\sigma \notin \Phi_i$ and $\neg\sigma \notin \Phi_i$. If it is complete, give a brief argument why. (1 point each for Φ_0 and Φ_1, 2 points for Φ_2, 3 points for Φ_3.)

Exercise 36 (8 points).

Consider the language of arithmetic $\mathcal{L} = \{\dot{+}, \dot{\times}, \dot{S}, \dot{0}, \dot{1}, \dot{<}, \dot{=}\}$ and its standard model

$$\mathbf{N} := \langle \mathbb{N}, +, \cdot, \mathrm{succ}, 0, 1, <, = \rangle.$$

(Here succ is the successor function $n \mapsto n + 1$.) The language of arithmetic allows to define formulas that describe the natural numbers:

$$\chi_n(x) :\hat{=} x \dot{=} \underbrace{\dot{S} \ldots \dot{S}}_{\text{n times}} \dot{0}$$

We say that a set of $\mathcal{L}$-sentences T is an *arithmetic* if $\mathbf{N} \models T$. Prove that every arithmetic has a model which is not isomorphic to $\mathbf{N}$.

Hint. Define an extension $\mathcal{L}^* := \mathcal{L} \cup \{\dot{c}\}$ of $\mathcal{L}$ where $\dot{c}$ is a constant symbol and look at the theory $T^* := T \cup \{\neg\chi_n(\dot{c})\,;\, n \in \mathbb{N}\}$. Prove that there is no value c of $\dot{c}$ such that $\langle \mathbf{N}, c \rangle$ is a model of T^*. Prove that T^* is consistent by using the compactness theorem. Use these two facts to prove the claim. (You may use that isomorphic models satisfy the same sentences.)