



Core Logic

2006/2007; 1st Semester
dr Benedikt Löwe

Homework Set # 12

Deadline: December 13th, 2006

Exercise 40 (6 points).

The ordinal ω_1^{CK} is sometimes called “the least admissible ordinal” and has an equivalent description in terms of an axiom system called “Kripke-Platek Set Theory” KP. Give a precise definition of ω_1^{CK} (2 points) and of KP (2 points) and give a brief (two to four sentences) description of the connection between the ordinal and KP (2 points).

Exercise 41 (5 points).

What is an Erdős number? This can either be a large cardinal notion (give a one-sentence description; 1 point) or a property of researchers (give a full recursive definition of “ X has Erdős number n ”; 2 points).

Compute the Erdős number of Johan van Benthem and Bill Gates (in both cases, give the shortest path witnessing the upper bound; 1 point each).

Exercise 42 (3 points).

Explain why Kripke models \mathbf{F} modelling the natural language notion of “it is allowed that” (i.e., $\mathbf{F} \models \Diamond\varphi$ means “ φ is allowed”) are not in general reflexive.

Exercise 43 (8 points).

Let $\langle \mathbf{M}, V \rangle$ be a Kripke model. We define

$$\text{not } \varphi := \Box \neg \varphi.$$

Let DN_0 (for “*duplex negatio*”) be $\text{not not } \varphi \rightarrow \varphi$ and DN_1 be $\varphi \rightarrow \text{not not } \varphi$.

- (1) Do DN_0 and DN_1 hold in the class of all reflexive, transitive frames (“S4-frames”; 2 points each)?
- (2) Do DN_0 and DN_1 hold in the class of all reflexive, symmetric, transitive frames (“S5-frames”; 2 points each)?