

Core Logic

1st Semester 2005/2006, period a & b

Dr Benedikt Löwe

Origins of Logic

- Greek mathematics
- Rhetoric: “Eristic” and “Sophistry”



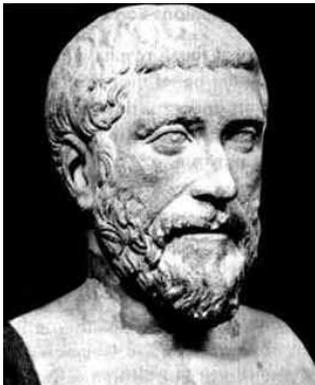
Greek mathematics.

- Pre-greek mathematics was not primarily concerned with proof, but more with computation. (Egyptians, Babylonians)
Geometry = measurement of the earth
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(Proclus, *In Primum Euclidis Elementorum Librum Commentarii*)



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- Mathematics built on proof:
 - Theaetetus (c.417-c.369 BC); student of Socrates
 - Euclid (c.325-c.265 BC); compilation of mathematical knowledge

Mathematical techniques.

- The axiomatic method.
 - Basic statements which are obviously true: axioms.
 - Rules of derivation.
 - Derivations from the axioms.
- Proof by contradiction

Mathematical techniques.

- Proof by contradiction

Claim. $\sqrt{2}$ is not a fraction of integers.

Suppose it were, then there are integers n and m without common divisor such that

$$\sqrt{2} = \frac{n}{m}.$$

But then

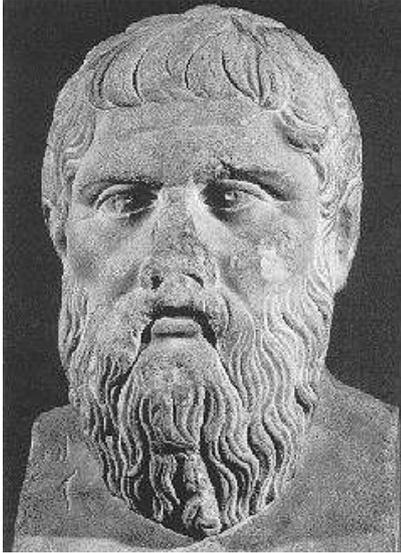
$$2m^2 = n^2.$$

In particular, n must be even. But then n^2 must be divisible by 4, and so m must be even. Contradiction.

Informal logic.

- The Dialectic method.
 - Proof by contradiction in mathematics.
 - Zeno of Elea (c.490-c.425 BC)
 - Socrates (469-399 BC; *elenchus*)
- Logic for “encounters”/“conversations”
 - Plato, *Euthydemus*
 - Aristotle, *Topics* and *Rhetoric*
 - Sophists
 - Public disputations according to rules for questioner and answerer
 - Megarians (next week)

Plato.



Plato (c.427-347 BC)

- Student and follower of Socrates until 399 B.C.
- 399-387 *BC*: Plato travels widely, including Italy and Sicily
- 387 *BC*: Plato founds the **Academy**
- 362 *BC*: Plato is invited to Sicily by Dionysios II.
- 347 *BC*: Plato dies and is succeeded by Speusippus

The Platonic Academy.

387 BC – 526 AD

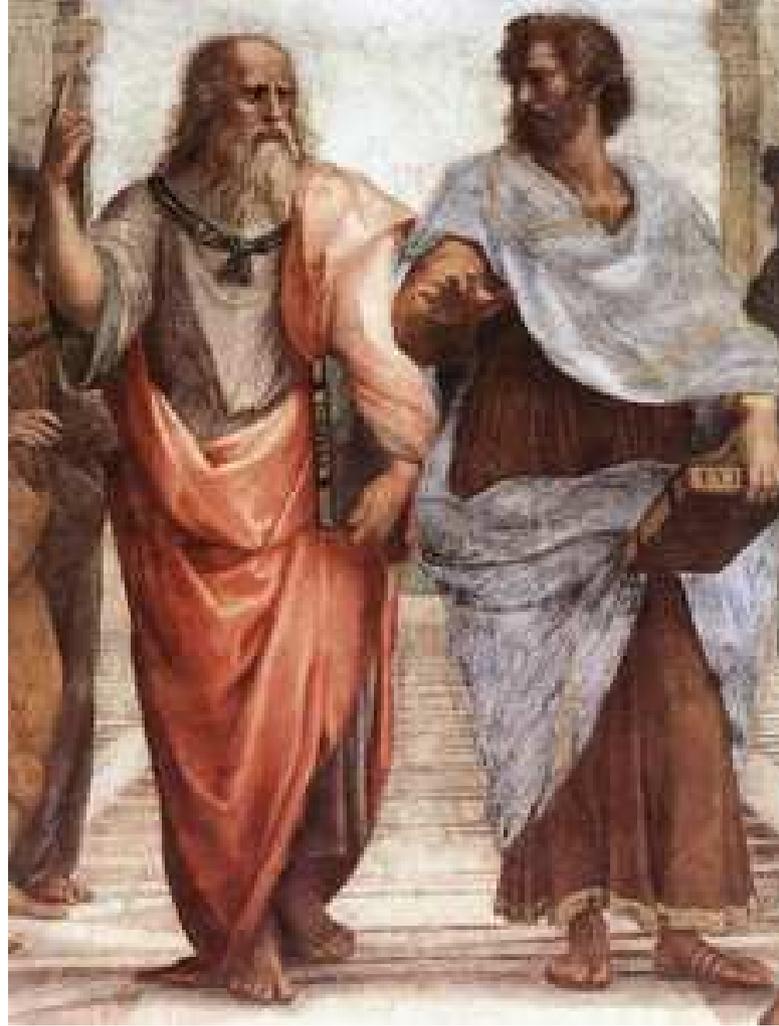
Academia was a public garden named after its donator *Academos*.

Μηδεις ἀγεωμέτριτος εἰσίτω μου τὴν στέγην

David Fowler, *The Mathematics of Plato's Academy: A New Reconstruction*

Members. Speusippus (347-339), Xenocrates (339-314), Polemo, Crates, Crantor, Arcesilaus (268-240), Lacydes, Evander, Hegesinus, Carneades, Clitomachus, and Philo ... and **Aristotle**.

Theoria et Praxis (1).



The School of Athens (Raffaello Sanzio; 1509)

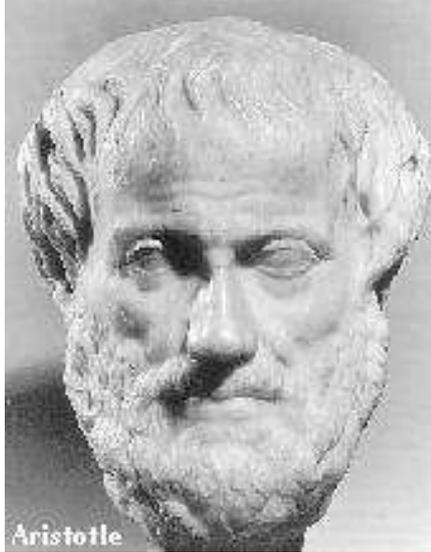
Theoria et Praxis (2).

[Uestium Philosophiae] in extremo margine Π Graecum, in supremo uero Θ legebatur intextum atque inter utrasque litteras in scalarum modum gradus quidam insigniti uidebantur, quibus ab inferiore ad superius elementum esset ascensus.

Boëthius, *Consolatio Philosophiae*
Book 1, Prosa 1

On the lowest border of [the garments of *Philosophia*] a Greek Π was embroidered, while on the highest a Θ could be read, and between both letters an ascent could be seen in the manner of stairs, by which you could move from the lower to the higher element.

Aristotle.



Aristotle (384-322 BC)

- 367 *BC*: Aristotle joins the Academy.
- 347 *BC*: Plato dies, Aristotle leaves Athens.
- 343-336 *BC*: Aristotle works at the court of Macedonia.
- 335 *BC*: Aristotle founds the *Lyceum* in Athens (Peripatetics).
- 323 *BC*: Alexander the Great dies, Aristotle retires to Chalcis.

Esoteric / exoteric.

Aristotle:

- **Esoteric works:** lecture notes and textbooks, designed for use within the Lyceum.
- **Exoteric works:** dialogues (modelled after the Platonic dialogues), designed for the general public.

“Plato’s unwritten doctrine”:

- Neoplatonism: Plotinus (204-270 AD)
- Porphyry (c.232-c.305 AD)
- [St. Augustine (354-430 AD)]
- Proclus (411-485 AD)

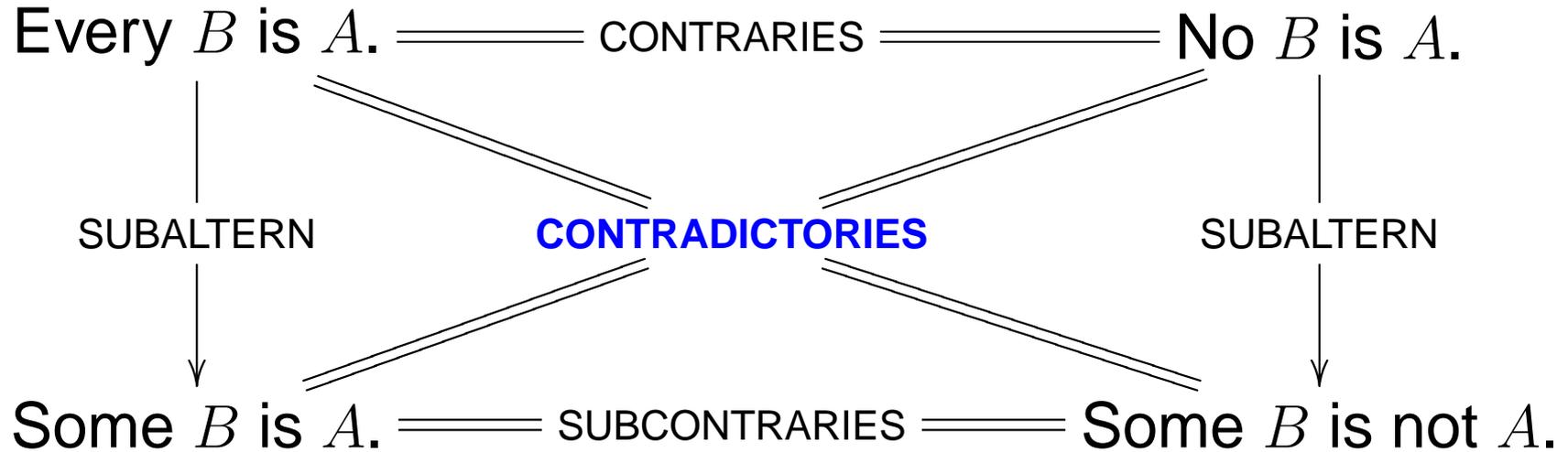
Aristotle's work on logic.

The Organon.

- **Categories:** Classification of types of predicates
- **On Interpretation** (*De interpretatione*): Basics of philosophy of language, subject-predicate distinction, Square of Oppositions
- **Prior Analytics:** Syllogistics
- **Posterior Analytics:** More on syllogistics
- **Topics:** Logic except for syllogistics
- **On Sophistical Refutations** (*De Sophisticis Elenchis*): Fallacies

The square of oppositions.

Aristotle, *De interpretatione*



- **Contradictory** propositions cannot both be true and they cannot both be false.
- **Contrary** propositions cannot both be true but can both be false.
- **Subcontrary** propositions cannot both be false but can both be true.
- A **subaltern** must be true if its **superaltern** is true, and the **superaltern** must be false if the **subaltern** is false.

The Categories.

Aristotle, *Categories*:
The ten categories (1b25).

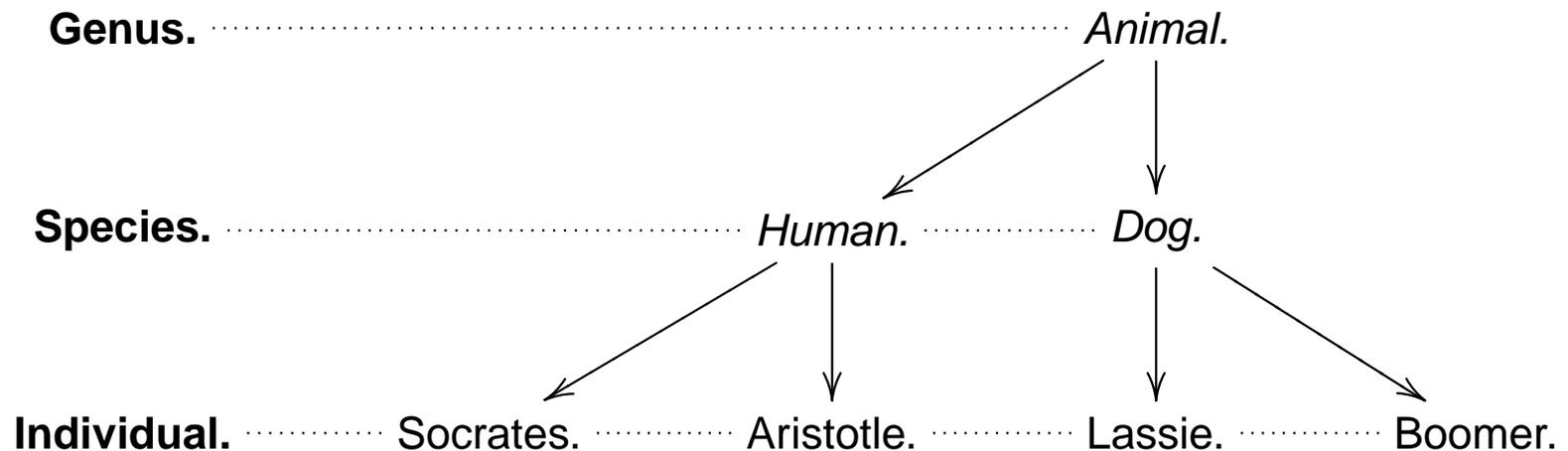
Substance	When
Quality	Position
Quantity	Having
Relation	Action
Where	Passion

The two ways of predication.

- *essential predication*: “Socrates is a human being”;
“*human* IS SAID OF Socrates”
- *accidental predication*: “Socrates is wise”; “*wisdom* IS
IN Socrates”

Essential predication.

- “essential”: You cannot deny the predicate without changing the meaning of the subject.
 - “*animal* IS SAID OF *human*”.
 - “*human* IS SAID OF Socrates”.
- IS SAID OF is a transitive relation.
- Related to the category tree:



Substances.

	NOT IN	IN
SAID OF	Universal substances <i>human, animal</i>	Universal accidents <i>wisdom</i>
NOT SAID OF	Particular substances Socrates, Aristotle	Particular accidents



- **Plato** (\uparrow). The **universal substances** are the (only) real things.
- **Aristotle** (\downarrow). Without the **particular substances**, nothing would exist.

Matter & Form.

- *Categories / De anima*: There are three kinds of substance: *matter*, *form* and the compound of the two.
- *Matter is potentiality; form is actuality.*
- **Aristotle in the *Metaphysics***: *Primary substances* cannot be compounds, not even of matter and form. Matter cannot be *primary*, therefore, the *primary substance* is the form.
- *Metaphysics Z (1037a6)*: “it is also clear that the soul is the primary substance, the body is matter, and man or animal is composed of the two as universal.”