UNIVERSITEIT VAN AMSTERDAM
INSTITUTE FOR LOGIC, LANGUAGE AND COMPUTATION

Core Logic

2004/2005; 1st Semester
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Homework Set # 4 *Deadline:* October 6th, 2004

We define a different formal system for proving valid moods. In the following, we use the letters t_{ij} for terms and the letters k_i stand for copulae. We write a mood in the form

$$\begin{array}{c} t_{11}\ k_1\ t_{12} \\ t_{21}\ k_2\ t_{22} \\ \hline t_{31}\ k_3\ t_{32}, \end{array}$$

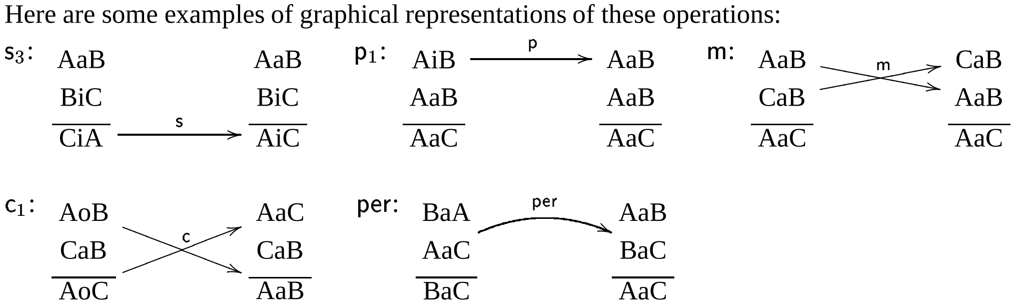
for example,

$$\begin{array}{c} \text{AaB} \\ \text{BaC} \\ \hline \text{AaC} \end{array}$$

for **Barbara**. We write M_i for $t_{i1}\ k_i\ t_{i2}$ and define some operations on moods.

- For $i \in \{1, 2, 3\}$, the operation s_i can only be applied if k_i is either 'i' or 'o'. In that case, s_i interchanges t_{i1} and t_{i2}.
- For $i \in \{1, 2, 3\}$, let p_i be the operation that changes k_i to its superaltern (if it has one).
- Let m be the operation that exchanges M_1 and M_2.
- For $i \in \{1, 2\}$, let c_i be the operation that first gives k_i and k_3 to their contradictories and then exchanges M_i and M_3.
- Let per be a permutation of the letters A, B, and C, applied to the mood. (It can be the identity.)

Here are some examples of graphical representations of these operations:

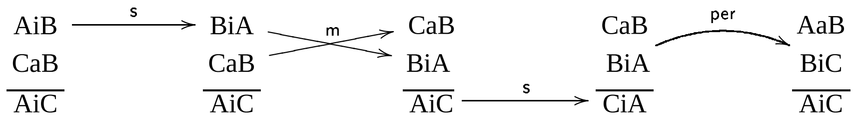


Given any set $\mathfrak{B}$ of "basic moods", a $\mathfrak{B}$-**proof** of a mood $M = M_1, M_2{:}M_3$ is a sequence $\langle \mathsf{o}_1, ..., \mathsf{o}_n \rangle$ of operations such that

- Only o_1 can be of the form c_1 or c_2 (but doesn't have to be).
- The sequence of operations, if applied to M, yields an element of $\mathfrak{B}$.

From the proof, we can derive a name for the valid mood via the medieval mnemonics. (Note that the use of s_i, p_i and c_i will add the corresponding consonant after the ith vowel.)

As an example, this is a proof of **Disamis**:



Exercise 8 (total of six points).

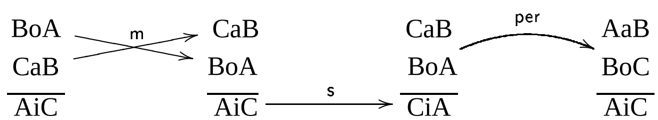
Let $\mathfrak{B}_{\mathrm{BCDF}}$ be the Aristotle's set of perfect moods **Barbara**, **Celarent**, **Darii**, and **Ferio**. Give $\mathfrak{B}_{\mathrm{BCDF}}$-proofs of **Camestres**, **Camenes** and **Bramantip** in the graphic representation given above (2 points each).

Exercise 9 (total of nine points).

Let $\mathfrak{B}_{\mathrm{GH}} := \{\mathbf{Giliri}, \mathbf{Halodri}\}$

where $\mathbf{Giliri}$ is $\begin{array}{c}\mathrm{AiB}\\ \mathrm{BiC}\\ \hline \mathrm{AiC}\end{array}$ and $\mathbf{Halodri}$ is $\begin{array}{c}\mathrm{AaB}\\ \mathrm{BoC}\\ \hline \mathrm{AiC}\end{array}$.

For example, the following is a $\mathfrak{B}_{\mathrm{GH}}$-proof:



Following the proof, the mood $\mathrm{BoA}, \mathrm{CaB}{:}\mathrm{AiC}$ could be called **Homalis**.

Give $\mathfrak{B}_{\mathrm{GH}}$-proofs in the graphic representation (2 points each) and find names consistent with the medieval mnemonics (1 point each) for the following three moods:

$$\begin{array}{c}\mathrm{BoA}\\ \mathrm{CiB}\\ \hline \mathrm{AiC}\end{array} \qquad \begin{array}{c}\mathrm{BiA}\\ \mathrm{CeB}\\ \hline \mathrm{AeC}\end{array} \qquad \begin{array}{c}\mathrm{AeB}\\ \mathrm{BiC}\\ \hline \mathrm{AeC}.\end{array}$$

Exercise 10 (total of ten points).

Let W be a nonempty set of states and $R \subseteq W \times W$ an accessibility relation. We say "state v is conceivable by anyone in state w" for wRv. Let X be a nonempty set of objects, and $E \subseteq W \times X$ a relation. We say "object x exists in state w" for wEx. For each $w \in W$, we have an order $<_w$ of X, and we say "in state w, object x is better than object y" for $y <_w x$.

We call $\langle W, R, X, E, \langle <_w\, ; \, w \in W \rangle \rangle$ an **ontological frame** if R is reflexive (*i.e.*, w is conceivable by anyone in state w), and the following principle *"Existence is better than nonexistence"* (EBN) holds:

(EBN) For all x, y and w, if wEx and $\neg wEy$, then $y <_w x$.

The central argument of Anselm's ontologic proof is "if something is such that nothing better can be conceived, then it must exist". Formulate this argument in the language of ontological frames and prove it (6 points).

Given an example of an ontological frame where there is no object "such that nothing better can be conceived" (4 points).