UNIVERSITEIT VAN AMSTERDAM
INSTITUTE FOR LOGIC, LANGUAGE AND COMPUTATION

Core Logic

2004/2005; 1st Semester
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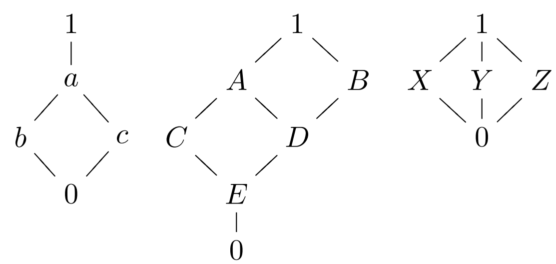
Homework Set # 11 *Deadline:* December 1st, 2004

Exercise 34 (11 points total).

Consider the following three lattices $\mathbf{L}_1$, $\mathbf{L}_2$ and $\mathbf{L}_3$:



For each of them define a (partial) unary function $-$ by "$-x$ is the greatest element y such that $x \wedge y = 0$ if this exists and is undefined otherwise". Determine $-x$ for all 17 elements in the three lattices (¼ point each). Determine whether $-$ is a total function on the three lattices (¼ point each). Does one the lattices satisfy $--x = x$ (give an argument; 3 points)? Does one of the lattices satisfy $---x = -x$ (give an argument; 3 points)?

Exercise 35 (8 points total).

Let $\mathbf{P} := \langle P, \leq \rangle$ be a **partial preorder** (*i.e.*, $\leq$ is a reflexive and transitive relation). For $x, y \in P$, define $x \equiv y$ by $x \leq y$ & $y \leq x$. Show that $\equiv$ is an equivalence relation (3 points). Let $D := P/\!\equiv$ be the set of $\equiv$-equivalence classes. For $\mathbf{d}, \mathbf{e} \in D$, define $\mathbf{d} \leq \mathbf{e}$ if and only if there are $x \in \mathbf{d}$ and $y \in \mathbf{e}$ such that $x \leq y$. Show that this is well-defined (2 points) and that $\langle D, \leq \rangle$ is a partial order (3 points).

Exercise 36 (6 points total).

Find out (and give an argument) whether the following clauses are satisfiable (2 points each):

(1) $(a \vee x \vee \neg p) \wedge (p \vee a \vee \neg x) \wedge (x \vee \neg x \vee a)$
(2) $(y \vee X) \wedge (\neg X \vee \neg y) \wedge (y \vee \neg X)$
(3) $(\alpha \vee \beta \vee \gamma) \wedge (\neg\gamma \vee \neg\beta \vee \neg\alpha) \wedge (\neg\gamma \vee \alpha \vee \beta)$