Universiteit van Amsterdam
Institute for Logic, Language and Computation

Recursion Theory

2003/2004; 1st Semester
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Homework Set # 7. Deadline: November 13th, 2003

Exercise # 1.

In analogy to the notion of Π^0_1-hardness, let's say a set H is Γ**-hard** if for all $A \in \Gamma$, we have $A \leq_{\mathrm{m}} H$ (where Γ is one of the Σ^0_n and Π^0_n). Let

$$S^*_2 := \{e\,;\, \exists y\, \forall x\, (\varphi^{(2)}_e(x,y) = 1)\}, \text{ and}$$
$$P^*_3 := \{e\,;\, \forall z\, \exists y\, \forall x\, (\varphi^{(3)}_e(x,y,z) = 1)\}.$$

Show that S^*_2 is Σ^0_2-hard and that P^*_3 is Π^0_3-hard.

Exercise # 2.

Recall from the lecture that if $T \subseteq \mathrm{Fml}$ (*i.e.*, T is a set of natural numbers coding formulae of the formal language of arithmetic) and $s := \langle s_0, \ldots, s_n \rangle$ is a proof in T, then we call

$$\#\, s := \prod_{i=0}^{n} p_i^{s_i}$$

the proof code of s, and we defined $\mathrm{Conseq}(T) := \{k \in \mathrm{Fml}_0\,;$ there is a proof $s := \langle s_0, \ldots, s_n \rangle$ in T and $i \leq n$ such that $k = s_i\}$. In class, we argued briefly that if T is a recursive set, then $\mathrm{Conseq}(T)$ is r.e.

Prove:

(1) If T is r.e., then $\mathrm{Conseq}(T)$ is r.e. as well.
(2) There is some T such that $T \not\leq_{\mathrm{m}} \mathrm{Conseq}(T)$.
Hint. Find $S \subseteq T$ such that $\mathrm{Conseq}(S) = \mathrm{Conseq}(T)$ where T is recursive and S is not r.e.