Mapping class group representations in combinatorial quantization

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Let $\Sigma_{g,n}$ be a compact oriented surface of genus g with n disks removed, and let H be a ribbon Hopf algebra, which we call the gauge algebra. The graph algebras $\mathcal{L}_{g,n}(H)$ have been introduced and studied around 1995 by Alekseev–Grosse–Schomerus and Buffenoir–Roche, under the name "combinatorial quantization". When $H = U_q(\mathfrak{g})$, q generic, the algebra $\mathcal{L}_{g,n}(U_q(\mathfrak{g}))$ is a quantization of the Atiyah–Bott–Goldman Poisson structure on the G-character variety of the surface, or equivalently, of the Fock–Rosly Poisson structure on the lattice gauge field theory with gauge group G, where G is a Lie group satisfying some conditions and $\mathfrak{g} = \mathrm{Lie}(G)$.

Moreover, Alekseev and Schomerus constructed projective representations of the mapping class group of $\Sigma_{g,n}$ using the algebras $\mathcal{L}_{g,n}(H)$. In the previously cited works, H was either semi-simple or a semi-simple truncation of a quantum group at a root of unity.

The results presented in this talk do not assume semi-simplicity of H. More precisely, I will assume that the gauge algebra H is finite-dimensional and factorizable (a technical condition), but not necessarily semi-simple, the guiding example being $\overline{U}_q(\mathfrak{sl}_2)$, the restricted quantum group associated to $\mathfrak{sl}_2(\mathbb{C})$. First, I will recall the origin, the definition and the properties of the algebras $\mathcal{L}_{g,n}(H)$. Then I will explain the construction of the projective representations of mapping class groups based on these algebras.

Finally, I will describe the case of the torus $\Sigma_{1,0}$ in great detail. I will show that the representation of the mapping class group $\mathrm{SL}_2(\mathbb{Z})$ is equivalent to that found by Lyubashenko and Majid. For $H = \overline{U}_q(\mathfrak{sl}_2)$, I will describe the representation explicitly on a suitable basis which was introduced by Gainutdinov–Tipunin and Arike.