Exercises in Algebraic Topology (master)

Prof. Dr. Birgit Richter

Summer term 2025

Exercise sheet no 3 due: 22nd of April 2025, 13:45h in H3

Thanks to the Easter break this sheet is a bit shorter than usual.

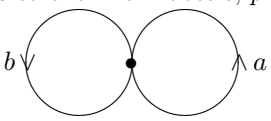
1 (Linear algebra) (3 + 1 points)
Compare the homology groups of $GL_n(\mathbb{R})$ and $O(n)$. What about $GL_n(\mathbb{C})$ and $U(n)$?

2 (Covering maps) (2 + 2 + 2 + 1 points)

(1) Let $p\colon \tilde{X} \to X$ be a covering map. We know that the induced map on fundamental groups is a monomorphism. Is that also true for $H_1(p)$?

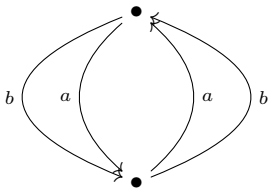
(2) Calculate the induced map on H_1 in the following examples:
a) the 2-sheeted covering from the torus to the Klein bottle, $p\colon T \to K$,

and for the two coverings of $\mathbb{S}^1 \vee \mathbb{S}^1$



indicated below:

b)



c)