Exercises in Algebraic Topology (master)

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Summer term 2024

Exercise sheet no 4 due: 26th of April 2024, 11:45h in H2

1 (Right complement?) (3 points)

Let $n \geqslant 0$ be any natural number. Can you find a pair of spaces (X_n, A_n) such that A_n is not the empty set and

$$H_0(X_n, A_n) \cong H_0(X_n \setminus A_n) \cong \mathbb{Z}^n?$$

2 (Too ugly?) (2 points)

What can you say about $H_1(\mathbb{R}, \mathbb{Q})$? Is it free abelian? Does it have torsion?

3 (Orientation) (2 + 2 points)

a) Take a closed orientable surface of genus g, F_g, and use excision to prove that $H_2(F_g, F_g \setminus \{x\}) \cong \mathbb{Z}$ for $x \in F_g$.

b) Do the same with the Möbius strip, M. Pick a generator $\mu_x \in H_2(M, M \setminus \{x\})$. What happens with the generator μ_x if you walk along the meridian of the Möbius strip?
`https://www.wikiart.org/en/m-c-escher/moebius-strip-ii`

4 (Mapping torus) (3 + 2 points)

Let $f, g \colon X \to Y$ be two continuous maps. The *mapping torus of f and g* is the space $T(f, g)$ defined as the quotient of $X \times [0,1] \sqcup Y$ by $(x, 0) \sim f(x)$ and $(x, 1) \sim g(x)$. (Important special cases are if f is the identity and g is a homeomorphism.)

a) Prove that there is a long exact sequence

$$\ldots \longrightarrow H_n(X) \xrightarrow{f_* - g_*} H_n(Y) \xrightarrow{i_*} H_n(T(f,g)) \xrightarrow{\delta} H_{n-1}(X) \xrightarrow{f_* - g_*} \ldots$$

b) Use this sequence to calculate the homology groups of the Klein bottle.