UNIVERSITEIT VAN AMSTERDAM
INSTITUTE FOR LOGIC, LANGUAGE AND COMPUTATION

Core Logic

2007/2008; 1st Semester
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Homework Set # 13 *Deadline:* December 12th, 2007

Exercise 44 (10 points).

In Recursion Theory, there is a concept called *the jump of a set* A, written A' ("A jump"). Find out what it is and give a precise definition in terms of the notions introduced in the lecture (4 points).
Are the following statements true or false (you don't have to give a proof, and you are allowed to use whatever literature is available):

- There is no fixed point of the jump-operation, i.e., a set A such that $A' = A$ (2 points).
- There is some A such that $(A')' = A'$ (2 points).
- There are A and B such that $A \neq B$ and $A' = B'$ (2 points).

Exercise 45 (4 points).

Explain why Kripke models $\mathbf{F}$ modelling the natural language notion of "it is allowed that" (*i.e.*, $\mathbf{F} \models \Diamond\varphi$ means "φ is allowed") are not in general reflexive.

Exercise 46 (8 points).

Let $\langle \mathbf{M}, V\rangle$ be a Kripke model. We define

$$\mathbf{not}\,\varphi := \Box\neg\varphi.$$

Let DN_0 (for "*duplex negatio*") be $\mathbf{not}\,\mathbf{not}\,\varphi \rightarrow \varphi$ and DN_1 be $\varphi \rightarrow \mathbf{not}\,\mathbf{not}\,\varphi$.

(1) Do DN_0 and DN_1 hold in the class of all reflexive, transitive frames ("$\mathbf{S4}$-frames"; 2 points each)?
(2) Do DN_0 and DN_1 hold in the class of all reflexive, symmetric, transitive frames ("$\mathbf{S5}$-frames"; 2 points each)?