# Reminder.

In syllogistics, all terms are nonempty.

**Barbari.** $A$a$B$, $B$a$C$: $A$i$C$.

Every unicorn is a white horse.
Every white horse is white.

---

There is a white unicorn.

# The perfect moods.

Τέλειον μὲν οὖν καλῶ συλλογισμὸν τὸν μηδενὸς ἄλλου προσδεόμενον παρὰ τὰ εἰλημμένα πρὸς τὸ φανῆναι τὸ ἀναγκαῖον. *(An.Pr. I.i)*

Aristotle discusses the first figure in *Analytica Priora* I.iv, identifies **Barbara**, **Celarent**, **Darii** and **Ferio** as *perfect* and then concludes

Δῆλον δὲ καὶ ὅτι πάντες οἱ ἐν αὐτῷ συλλογισμοὶ τέλειοί εἰσι ... καλῶ δὲ τὸ τοιοῦτον σχῆμα πρῶτον. *(An.Pr. I.iv)*

# Axioms of Syllogistics.

So the Axioms of Syllogistics according to Aristotle are:

**Barbara.** $A$a$B$, $B$a$C$ : $A$a$C$
**Celarent.** $A$e$B$, $B$a$C$: $A$e$C$
**Darii.** $A$a$B$, $B$i$C$ : $A$i$C$
**Ferio.** $A$e$B$, $B$i$C$: $A$o$C$

# Simple and accidental conversion.

- Simple (*simpliciter*).
  - $X\mathsf{i}Y \rightsquigarrow Y\mathsf{i}X$.
  - $X\mathsf{e}Y \rightsquigarrow Y\mathsf{e}X$.
- Accidental (*per accidens*).
  - $X\mathsf{a}Y \rightsquigarrow X\mathsf{i}Y$.
  - $X\mathsf{e}Y \rightsquigarrow X\mathsf{o}Y$.

# Syllogistic proofs.

A syllogistic proof is a sequence $\langle p_0, p_1, p_2, ..., p_n \rangle$ of categorical propositions such that for each $t > 1$,

- *either* there are $i, j < t$ such that $p_i, p_j : p_t$ is an instance of Barbara, Celarent, Darii or Ferio,
- *or* there is some $i < t$ such that $p_t$ is the result of converting $p_i$ according to one of the four conversion rules.

**Example 1.**

(0) $A$a$B$
(1) $C$i$B$
(2) $B$i$C$, (simple i-conversion from (1))
(3) $A$i$C$, (**Darii** from (0) and (2).)

# Syllogistic proofs.

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**Example 2.**

(0) $A$i$B$
(1) $C$a$B$
(2) $B$i$A$, (simple i-conversion from (0))
(3) $C$i$A$, (**Darii** from (1) and (2))
(4) $A$i$C$, (simple i-conversion from (3))

# Proving valid moods directly.

For a given mood $\mu$, a syllogistic proof $\langle p_0, p_1, p_2, ..., p_n\rangle$ is called a direct proof of $\mu$ if $p_0$ is the major premiss of $\mu$, $p_1$ is the minor premiss of $\mu$ and $p_n$ is the conclusion of $\mu$.

**Example 1**, $\langle A$a$B$, $C$i$B$, $B$i$C$, $A$i$C\rangle$ is a proof of **Datisi**.

**Example 2**, $\langle A$i$B$, $C$a$B$, $B$i$A$, $C$i$A$, $A$i$C\rangle$ is a proof of **Disamis**.

# Indirect syllogistic proof (1).

A indirect syllogistic proof is a sequence $\langle p_0, p_1, p_2, ..., p_n\rangle$ of categorical propositions such that for each $t > 2$,

- *either* there are $i, j < t$ such that $p_i, p_j : p_t$ is an instance of Barbara, Celarent, Darii or Ferio,
- *or* there is some $i < t$ such that $p_t$ is the result of converting $p_i$ according to one of the four conversion rules.

**Example 3.**

(0) $A$o$B$
(1) $C$a$B$
(2) *$A$a$C$
(3) $A$a$B$, (**Barbara** from (1) and (2))

# Indirect syllogistic proof (2).

For a given mood $\mu$, an indirect syllogistic proof $\langle p_0, p_1, p_2, ..., p_n\rangle$ is called a indirect proof of $\mu$ if $p_0$ is the major premiss of $\mu$, $p_1$ is the minor premiss of $\mu$, the contradictory of the conclusion of $\mu$ occurs in the sequence, and $p_n$ is the contradictory of one of the premises of $\mu$.

$\langle A$o$B$, $C$a$B$, *$A$a$C$, $A$a$B\rangle$ is a proof of **Bocardo**.

# Mnemonics (1).

*Bárbara, Célarént, Darií, Ferióque prióris,*
*Césare, Cámestrés, Festíno, Baróco secúndae.*
*Tértia Dáraptí, Disámis, Datísi, Felápton,*
*Bocárdo, Feríson habét. Quárta ínsuper áddit*
*Brámantíp, Camenés, Dimáris, Fesápo, Fresíson.*

“These words are more full of meaning than any that were ever made.” (Augustus de Morgan)

# Mnemonics (2).

- The first letter indicates to which one of the four perfect moods the mood is to be reduced: 'B' to Barbara, 'C' to Celarent, 'D' to Darii, and 'F' to Ferio.
- The letter 's' after the first or second vowel indicates that the corresponding premiss has to be simply converted.
- The letter 'p' after the first or second vowel indicates that the corresponding premiss has to be accidentally converted ("*per accidens*").
- The letter 's' after the third vowel indicates that the conclusion will be gained by simple conversion.
- The letter 'p' after the third vowel indicates that the conclusion will be gained by accidental conversion ("*per accidens*").
- The letter 'c' after the first or second vowel indicates that the mood has to be proved indirectly by proving the contradictory of the corresponding premiss.
- The letter 'm' indicates that the premises have to be interchanged ("moved").
- All other letters have only aesthetic purposes.

# A metatheorem.

Let $\mathrm{BCDF}$ be the full syllogistic system as described above. If $\mu$ is a mood, we write $\mathrm{BCDF} \vdash \mu$ if there is either a direct or an indirect proof of $\mu$. We call a premiss negative if it has either 'e' or 'o' as copula.

**Theorem** (Aristotle). If $\mu$ is a mood with two negative premises, then

$$\mathrm{BCDF} \nvdash \mu.$$

# Proof (1).

- Towards a contradiction, let $\langle p_0, \ldots, p_n \rangle$ be a proof of $\mu$. We know that $p_0$ and $p_1$ are negative premises, and that $p_0$ contains the terms $A$ and $B$ and $p_1$ contains the terms $B$ and $C$.
- **Case 1.** The proof is a direct proof. Then $p_n$ contains the terms $A$ and $C$.
- Note that none of the conversion rules can change the set of terms in a proposition, so some step in the proof must be an application of a perfect syllogism.
- Let $k$ be the first application of a perfect syllogism, *i.e.*, there are $i, j < k$ such that $p_i, p_j : p_k$ is either **Barbara**, **Celarent**, **Darii** or **Ferio**.

# Proof (2).

$k$ is least such that $p_i, p_j : p_k$ is a perfect syllogism.

- Since $k$ is least, all $p_m$ with $m < k$ must have been constructed from $p_0$ and $p_1$ by iterated application of conversion rules.
- Conversion rules can never make a negative proposition into a positive one.
- Ergo: for all $m < k$, $p_m$ is a negative proposition. In particular, this is true for $p_i$ and $p_j$.
- But no perfect syllogism has two negative premises. Contradiction! So the tentative proof was not direct.

# Proof (3).

$\langle p_0, \ldots, p_n \rangle$ is a proof of $\mu$, but not a direct proof.

- **Case 2.** So the proof must be an indirect proof, i.e., $p_2$ is the contradictory of the conclusion of $\mu$ and $p_n$ is the contradictory of one of the premises of $\mu$. (So, $p_n$ is a positive proposition.)
- This means that $p_2$ contains the terms $A$ and $C$, and $p_n$ contains either $A$ and $B$ or $B$ and $C$. Without loss of generality, let's assume that it contains $A$ and $B$.
- Let $k$ be the least number such that $p_k$ is a positive proposition with the terms $A$ and $B$.
- Since conversions cannot make a negative proposition positive, there must be $i, j < k$ such that $p_i, p_j : p_k$ is a perfect syllogism.

# Proof (4).

$k$ is least such that $p_k$ is a positive proposition with the terms $A$ and $B$. $p_i, p_j : p_k$ is a perfect syllogism.

- The only perfect syllogisms with positive conclusions are **Barbara** and **Darii**, but they require two positive premises, so $p_i$ and $p_j$ are positive.
- Without loss of generality, let $p_i$ have the terms $B$ and $C$. Again, conversions cannot make negative propositions positive, so there must be $i_0, i_1 < i$ such that $p_{i_0}, p_{i_1} : p_i$ is a perfect syllogism.
- As above, $p_{i_0}$ and $p_{i_1}$ must be positive.
- One of them (say, $p_{i_0}$) has the terms $A$ and $B$. Contradiction to the choice of $k$.

# Other metatheoretical results.

- If $\mu$ has two particular premises (i.e., with copulae ‘i’ or ‘o’), then $\mathrm{BCDF} \not\vdash \mu$ (**Exercise 7**).
- If $\mu$ has a positive conclusion and one negative premiss, then $\mathrm{BCDF} \not\vdash \mu$.
- If $\mu$ has a negative conclusion and one positive premiss, then $\mathrm{BCDF} \not\vdash \mu$.
- If $\mu$ has a universal conclusion (i.e., with copula ‘a’ or ‘e’) and one particular premiss, then $\mathrm{BCDF} \not\vdash \mu$.

# Aristotelian modal logic.

## Modalities.

- $\mathbf{A}p \doteq$ "$p$" (no modality, "assertoric").
- $\mathbf{N}p \doteq$ "necessarily $p$".
- $\mathbf{P}p \doteq$ "possibly $p$" (equivalently, "not necessarily not $p$").
- $\mathbf{C}p \doteq$ "contingently $p$" (equivalently, "not necessarily not $p$ and not necessarily not $p$").

Every (assertoric) mood $p, q : r$ represents a modal mood $\mathbf{A}p, \mathbf{A}q : \mathbf{A}r$. For each mood, we combinatorially have $4^3 = 64$ modalizations, i.e., $256 \times 64 = 16384$ modal moods.

# Modal conversions.

- **Simple.**
  - $\mathbf{N}X\mathrm{e}Y \rightsquigarrow \mathbf{N}Y\mathrm{e}X$
  - $\mathbf{N}X\mathrm{i}Y \rightsquigarrow \mathbf{N}Y\mathrm{i}X$
  - $\mathbf{C}X\mathrm{e}Y \rightsquigarrow \mathbf{C}Y\mathrm{e}X$
  - $\mathbf{C}X\mathrm{i}Y \rightsquigarrow \mathbf{C}Y\mathrm{i}X$
  - $\mathbf{P}X\mathrm{e}Y \rightsquigarrow \mathbf{P}Y\mathrm{e}X$
  - $\mathbf{P}X\mathrm{i}Y \rightsquigarrow \mathbf{P}Y\mathrm{i}X$
- **Accidental.**
  - $\mathbf{N}X\mathrm{a}Y \rightsquigarrow \mathbf{N}X\mathrm{i}Y$
  - $\mathbf{C}X\mathrm{a}Y \rightsquigarrow \mathbf{C}X\mathrm{i}Y$
  - $\mathbf{P}X\mathrm{a}Y \rightsquigarrow \mathbf{P}X\mathrm{i}Y$
  - $\mathbf{N}X\mathrm{e}Y \rightsquigarrow \mathbf{N}X\mathrm{o}Y$
  - $\mathbf{C}X\mathrm{e}Y \rightsquigarrow \mathbf{C}X\mathrm{o}Y$
  - $\mathbf{P}X\mathrm{e}Y \rightsquigarrow \mathbf{P}X\mathrm{o}Y$
- **Relating to the symmetric nature of contingency.**
  - $\mathbf{C}X\mathrm{i}Y \rightsquigarrow \mathbf{C}X\mathrm{e}Y$
  - $\mathbf{C}X\mathrm{e}Y \rightsquigarrow \mathbf{C}X\mathrm{i}Y$
  - $\mathbf{C}X\mathrm{a}Y \rightsquigarrow \mathbf{C}X\mathrm{o}Y$
  - $\mathbf{C}X\mathrm{o}Y \rightsquigarrow \mathbf{C}X\mathrm{a}Y$
- $\mathbf{N}X\mathrm{x}Y \rightsquigarrow \mathbf{A}X\mathrm{x}Y$
  (Axiom $\mathbf{T}$: $\Box\varphi \to \varphi$)

# Modal axioms.

What are the “perfect modal syllogisms”?

- Valid assertoric syllogisms remain valid if $\mathbf{N}$ is added to all three propositions.

  **Barbara** $(A\mathrm{a}B, B\mathrm{a}C{:}A\mathrm{a}C) \rightsquigarrow$ **NNN Barbara** $(\mathbf{N}A\mathrm{a}B, \mathbf{N}B\mathrm{a}C{:}\mathbf{N}A\mathrm{a}C)$.

  First complications in the arguments for **Bocardo** and **Baroco**.

- By our conversion rules, the following can be added to valid assertoric syllogisms:
  - $\mathbf{NNA}$,
  - $\mathbf{NAA}$,
  - $\mathbf{ANA}$.
- Anything else is problematic.

# The “two Barbaras”.

| **NAN Barbara** | **ANN Barbara** |
|---|---|
| $\mathbf{N}A\mathrm{a}B$ | $\mathbf{A}A\mathrm{a}B$ |
| $\mathbf{A}B\mathrm{a}C$ | $\mathbf{N}B\mathrm{a}C$ |
| $\mathbf{N}A\mathrm{a}C$ | $\mathbf{N}A\mathrm{a}C$ |

From the modern point of view, both modal syllogisms are invalid, yet Aristotle claims that **NAN Barbara** is valid, but **ANN Barbara** is not.

# *De dicto* versus *De re*.

We interpreted $\mathbf{N}A\mathrm{a}B$ as

“The statement ‘$A\mathrm{a}B$’ is necessarily true.’

(*De dicto* interpretation of necessity.)

Alternatively, we could interpret $\mathbf{N}A\mathrm{a}B$ *de re* (Becker 1933):

“Every $B$ happens to be something which is necessarily an $A$.”

# Aristotelian temporal logic: the sea battle

According to the square of oppositions, exactly one of“it is the case that $p$” and “it is not the case that $p$” is true.

*Either* “it is the case that there will be a sea battle tomorrow”

*or* “it is not the case that there will be a sea battle tomorrow”.

Problematic for existence of free will, and for Aristotelian metaphysics.

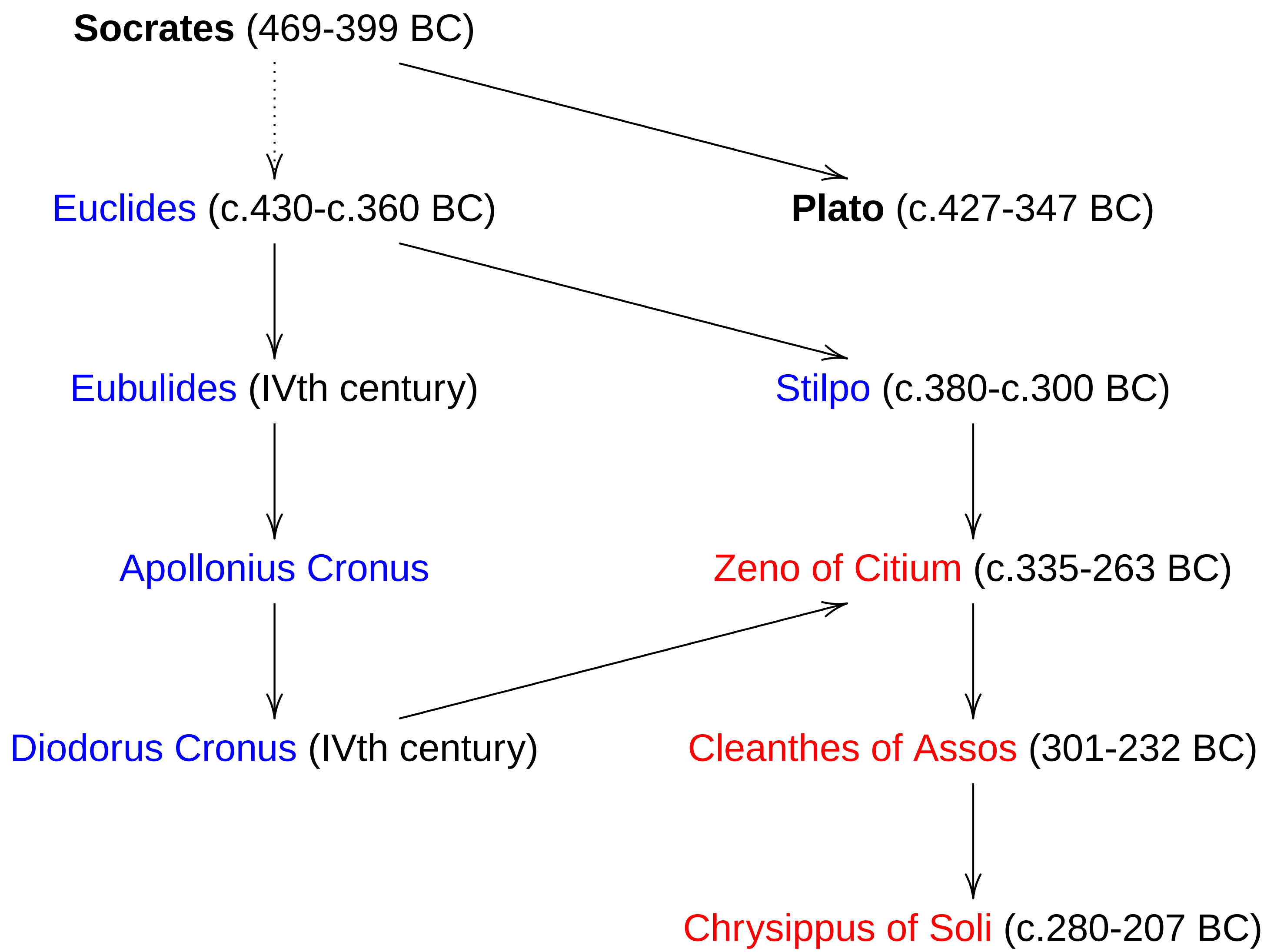
# The Master argument.

Diodorus Cronus (IVth century BC).

- Assume that $p$ is not the case.
- In the past, "It will be the case that $p$ is not the case" was true.
- In the past, "It will be the case that $p$ is not the case" was necessarily true.
- Therefore, in the past, "It will be the case that $p$" was impossible.
- Therefore, $p$ is not possible.

*Ergo:* Everything that is possible is true.

# Megarians and Stoics.



# Eubulides.

- Strongly opposed to Aristotle.
- Source of the “seven Megarian paradoxes”, among them the *Liar*.
  - *The Liar* is attributed to Epimenides the Cretan (VIIth century BC); (*Titus 1:12*).
  - **Aulus Gellius**, *Noctes Atticae*.
    Alessandro **Garcea**, Paradoxes in Aulus Gellius, **Argumentation** 17 (2003), p. 87-98
- Graham **Priest**, The Hooded Man, **Journal of Philosophical Logic** 31 (2002), p. 445-467

# The seven Megarian paradoxes.

- *The Liar.* “Is the man a liar who says that he tells lies?”
- *The concealed man.* “Do you know this man who is concealed? If you do not, you do not know your own father; for he it is who is concealed.”
- *The hooded man.* “You say that you know your brother. Yet that man who just came in with his head covered is your brother and you did not know him.”
- *Electra.* “Electra sees Orestes : she knows that Orestes is her brother, but does not know that the man she sees is Orestes; therefore she does know, and does not know, her brother at the same time.”
- *The Sorites / the heap.* “One grain of wheat does not make a heap. Adding one grain of wheat doesn’t make a heap.”
- *The bald one.* “Pulling one hair out of a man’s head will not make him bald, nor two, nor three, and so on till every hair in his head is pulled out.”
- *The horned one.* You have what you have not lost. You have not lost horns, therefore you have horns.

# ***Quarternio terminorum.***

I know my brother.

I do not know the hooded man.

The hooded man is my brother.

~~I do not know my brother.~~

know: "I know a lot about $X$."

know: "I recognize $X$ at sight (regardless of whether $X$ is hooded or not)."

Every metal is a chemical element.

Brass is a metal.

~~Brass is a chemical element.~~

# More shortcomings of syllogistics.

**Syllogistics** is finitary and cannot deal with very simple propositional connectives:

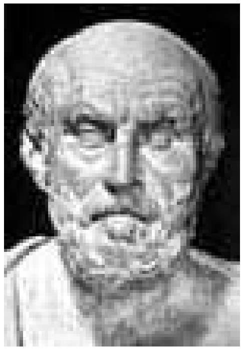
Every human being is a man or a woman.

Every man is mortal.

Every woman is mortal.

*Ergo...* every human being is mortal.

# Stoic Logic.



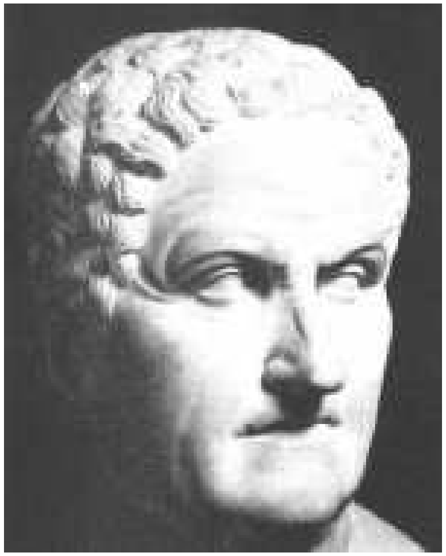
**Chrysippus of Soli** (c.280-207 BC)

- 118 works on logic,
- seven books on *the Liar*,
- inventor of propositional logic,
- nonstandard view of modal logic ("the impossible can follow from the possible").

  Harry **Ide**, Chrysippus's response to Diodorus's master argument, **History and Philosophy of Logic** 13 (1992), p. 133-148.

# Late antiquity.

- Galen (129-216)



*Galen of Pergamum*
(129-216)
Court Physician to Marc Aurel
*Introduction to Dialectics*
(rediscovered in XIXth century)

# Late antiquity.

- Galen (129-216)
- Augustine (354-430)


*(Sanctus) Aurelius Augustinus*
(354-430)
*doctor ecclesiae*

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- Galen (129-216)
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- Galen (129-216)
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- Boëthius (c.475-524)
- Cassiodorus (c.490-c.585)

*Flavius Magnus Aurelius Cassiodorus Senator*
(c.490-c.585)
**Main work:** *Institutiones*

# Late antiquity.

- Galen (129-216)
- Augustine (354-430)
- Boëthius (c.475-524)
- Cassiodorus (c.490-c.585)
- Isidore of Seville (c.560-636)

*(Sanctus) Isidorus Hispalensis*
(c.560-636)
**Main work:** *Etymologiae*

Patron Saint of the Internet

# Boëthius.

*Anicius Manlius Severinus Boëthius*
(c.475-524)

“The last of the Roman philosophers, and the first of the scholastic theologians" (Martin Grabmann)