Origins of Logic

- Greek mathematics
- Rhetoric: “Eristic” and “Sophistry”

Greek mathematics.

- Pre-greek mathematics was not primarily concerned with proof, but more with computation. (Egyptians, Babylonians)
  Geometry = measurement of the earth
- Thales of Miletus (c.625-c.546 BC): the first proof
  (Proclus, *In Primum Euclidis Elementorum Librum Commentarii*)
- Pythagoras (c.569-c.475 BC)
- Mathematics built on proof:
  - Theaetetus (c.417-c.369 BC); student of Socrates
  - Euclid (c.325-c.265 BC); compilation of mathematical knowledge

Mathematical techniques.

- Proof by contradiction

Claim. $\sqrt{2}$ is not a fraction of integers.

Suppose it were, then there are integers n and m without common divisor such that

$$\sqrt{2} = \frac{n}{m}.$$

But then

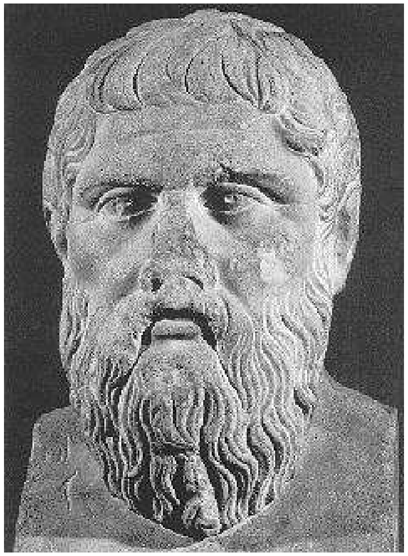
$$2m^2 = n^2.$$

In particular, n must be even. But then n^2 must be divisible by 4, and so m must be even. Contradiction.

Informal logic.

- The Dialectic method.
  - Proof by contradiction in mathematics.
  - Zeno of Elea (c.490-c.425 BC)
  - Socrates (469-399 BC; *elenchus*)
- Logic for “encounters”/“conversations”
  - Plato, *Euthydemus*
  - Aristotle, *Topics*
  - Sophists
  - Public disputations according to rules for questioner and answerer
  - Megarians (next week)

Plato.



Plato (c.427-347 BC)

- Student and follower of Socrates until 399 B.C.
- *399-387 BC:* Plato travels widely, including Italy and Sicily
- *387 BC:* Plato founds the **Academy**
- *362 BC:* Plato is invited to Sicily by Dionysios II.
- *347 BC:* Plato dies and is succeeded by Speusippus

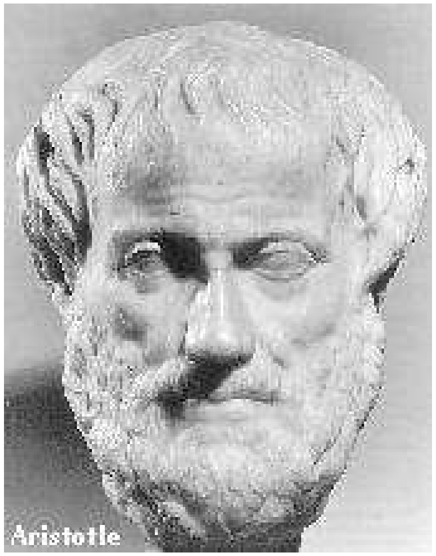
The Platonic Academy.

387 BC – 526 AD
Academia was a public garden named after its donator *Academus*.

David Fowler, The Mathematics of Plato's Academy: A New Reconstruction

Members. Speusippus, Xenocrates, Polemo, Crates, Crantor, Arcesilaus, Lacydes, Evander, Hegesinus, Carneades, Clitomachus, and Philo ... and **Aristotle**.

Aristotle.



Aristotle (384-322 BC)

- *367 BC*: Aristotle joins the Academy.
- *347 BC*: Plato dies, Aristotle leaves Athens.
- *343-336 BC*: Aristotle works at the court of Macedonia.
- *335 BC*: Aristotle founds the *Lyceum* in Athens (Peripatetics).
- *323 BC*: Alexander the Great dies, Aristotle retires to Chalcis.

Esoteric / exoteric.

Aristotle:

- **Esoteric works**: lecture notes and textbooks, designed for use within the Lyceum.
- **Exoteric works**: dialogues (modelled after the Platonic dialogues), designed for the general public.

“Plato’s unwritten doctrine”:

- Neoplatonism: Plotinus (204-270 AD)
- Porphyry (c.232-c.305 AD)
- [St. Augustine (354-430 AD)]
- Proclus (411-485 AD)

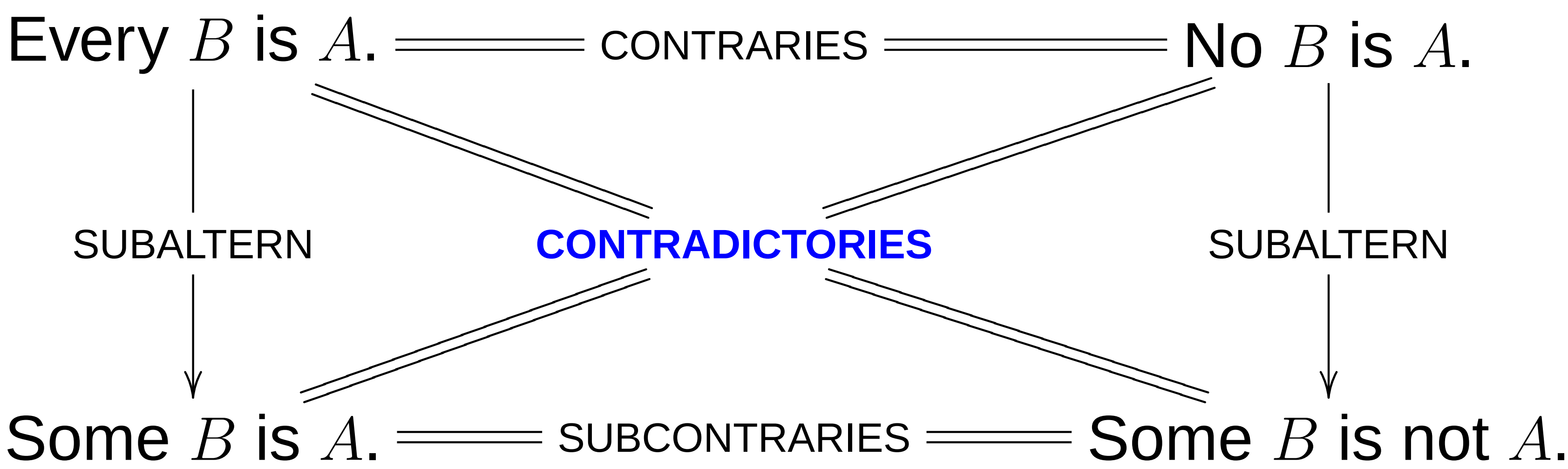
Aristotle's work on logic.

The **Organon**.

- **Categories**: Classification of types of predicates
- **On Interpretation***(De interpretatione): Basics of philosophy of language, subject-predicate distinction, Square of Oppositions*
- **Prior Analytics**: Syllogistics
- **Posterior Analytics**: More on syllogistics
- **Topics**: Logic except for syllogistics
- **On Sophistical Refutations** *(De Sophisticis Elenchis)*: Fallacies

The square of oppositions.

Aristotle, *De interpretatione*



- **Contradictory** propositions cannot both be true and they cannot both be false.
- Contrary propositions cannot both be true but can both be false.
- Subcontrary propositions cannot both be false but can both be true.
- A subaltern must be true if its superaltern is true, and the superaltern must be false if the subaltern is false.

The most famous syllogism.

Every man is mortal.
Socrates is a man.

Socrates is mortal.

Proper name

A more typical syllogism.

Every animal is mortal.
Every man is an animal.

Every man is mortal.

Every B is an A.
Every C is a B.

Every C is an A.

“Barbara”

“a valid mood”
mood = *modus*

Another valid mood.

Every philosopher is mortal.
Some teacher is a philosopher.

Some a teacher is mortal.

Every B is an A.
Some C is a B.

Some C is an A.

“Darii”

A similar but invalid mood.

“Darii”

Every B is an A.
Some C is a B.

Some C is an A.

Every A is a B.
Some C is a B.

Some C is an A.

Every philosopher is mortal.
Some teacher is mortal.

~~Some teacher is a philosopher.~~

Yet another very similar mood.

"Darii"	The invalid mood	**"Datisi"**
Every B is an A.	Every A is a B.	Every B is a A.
Some C is a B.	Some C is a B.	Some B is a C.
Some C is an A.	Some C is an A.	Some C is an A.

"Some C is a B" and "Some B is a C"
are intuitively equivalent.

A first conversion rule.

This yields a simple formal (syntactical) conversion rule:

“Some X is a Y”

can be converted to

“Some Y is an X.”

This rule is validity-preserving and syntactical.

Back to *Darii* and *Datisi*.

"Darii"

Every B is an A.
Some C is a B.

Some C is an A.

"Datisi"

Every B is a A.
Some B is a C.

Some C is an A.

Simple Conversion

"Some X is a Y" $\rightsquigarrow$ "Some Y is an X"

Methodology of Syllogistics.

- Start with a list of obviously valid moods (perfect syllogisms $\cong$ “axioms”)...
- ...and a list of conversion rules,
- derive all valid moods from the perfect syllogisms by conversions,
- and find counterexamples for all other moods.

Notation (1).

Syllogistics is a term logic, not propositional or predicate logic.

We use capital letters A, B, and C for terms, and sometimes X and Y for variables for terms.

Terms (*termini*) form part of a categorical proposition. Each categorical proposition has two terms:
a subject and a predicate, connected by a copula.

Every B is an A.

Notation (2).

There are four copulae:

- The universal affirmative: Every — is a —. a
- The universal negative: No — is a —. e
- The particular affirmative: Some — is a —. i
- The particular negative: Some — is not a —. o

Every B is an A. $\rightsquigarrow$ AaB
No B is an A. $\rightsquigarrow$ AeB
Some B is an A. $\rightsquigarrow$ AiB
Some B is not an A. $\rightsquigarrow$ AoB

Contradictories: a–o & e–i.

Notation (3).

	Every B is an A	Aa B
Barbara	Every C is a B	Ba C
	Every C is an A	Aa C

Each syllogism contains three terms and three categorial propositions. Each of its categorial propositions contains two of its terms. Two of the categorial propositions are premises, the other is the conclusion.
The term which is the predicate in the conclusion, is called the major term, the subject of the conclusion is called the minor term, the term that doesn't occur in the conclusion is called the middle term.

Notation (4).

	Every B is an A	A a B
Barbara	Every C is a B	B a C
	Every C is an A	A a C

Major term / Minor term / Middle term

Only one of the premises contains the major term. This one is called the major premise, the other one the minor premise.

Ist Figure

$$A — B, B — C : A — C$$

IInd Figure

$$B — A, B — C : A — C$$

IIIrd Figure

$$A — B, C — B : A — C$$

IVth Figure

$$B — A, C — B : A — C$$

Notation (5).

If you take a figure, and insert three copulae, you get a mood.

Ist Figure: A a B , B a C : A a C

Barbara

Combinatorics of moods.

With four copulae and three slots, we get

$$4^3 = 64$$

moods from each figure, *i.e.*, $4 \times 64 = 256$ in total.
Of these, 24 have been traditionally seen as valid.

A	a	B	,	B	i	C	:	A	i	C		
D	**a**	r			**i**				**i**		$\leadsto$	**Darii**
A	a	B	,	C	i	B	:	A	i	C		
D	**a**	t			**i**	s			**i**		$\leadsto$	**Datisi**

The 24 valid moods (1).

Ist figure	AaB	,	BaC	:	AaC	**Barbara**
	AeB	,	BaC	:	AeC	**Celarent**
	AaB	,	BiC	:	AiC	**Darii**
	AeB	,	BiC	:	AoC	**Ferio**
	AaB	,	BaC	:	AiC	**Barbari**
	AeB	,	BaC	:	AoC	**Celaront**
IInd figure	BeA	,	BaC	:	AeC	**Cesare**
	BaA	,	BeC	:	AeC	**Camestres**
	BeA	,	BiC	:	AoC	**Festino**
	BaA	,	BoC	:	AoC	**Baroco**
	BeA	,	BaC	:	AoC	**Cesaro**
	BaA	,	BeC	:	AoC	**Camestrop**

The 24 valid moods (2).

IIIrd figure	AaB	,	CaB	:	AiC	**Darapti**
	AiB	,	CaB	:	AiC	**Disamis**
	AaB	,	CiB	:	AiC	**Datisi**
	AeB	,	CaB	:	AoC	**Felapton**
	AoB	,	CaB	:	AoC	**Bocardo**
	AeB	,	CiB	:	AoC	**Ferison**
IVth figure	BaA	,	CaB	:	AiC	**Bramantip**
	BaA	,	CeB	:	AeC	**Camenes**
	BiA	,	CaB	:	AiC	**Dimaris**
	BeA	,	CaB	:	AoC	**Fesapo**
	BeA	,	CiB	:	AoC	**Fresison**
	BaA	,	CeB	:	AoC	**Camenop**