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Geometric Topology

Problem Set 1

1. Prove that the subset

$$Q := \{\mathbf{z} = (z_1, \dots, z_n) \in \mathbb{C}^n \mid \sum_{j=1}^{n} z_j^2 = 1\} \subset \mathbb{C}^n \cong \mathbb{R}^{2n}$$

is diffeomorphic to the tangent bundle of S^{n-1}.

2. Prove or disprove:

a) There is an immersion of the punctured torus $S^1 \times S^1 \setminus \{pt\}$ into $\mathbb{R}^2$.

b) Any finite product of spheres admits an embedding of codimension 1 into euclidean space.

3. Prove that every closed connected 1-dimensional manifold is diffeomorphic to $S^1 \subseteq \mathbb{R}^2$.

4. Let $U \subset \mathbb{R}^n$ be a connected open subset, and let $p : U \to U$ be a smooth map such that $p \circ p = p$. Prove that the subset $F \subset U$ of fixed points of p forms a smooth submanifold of $\mathbb{R}^n$.