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Winter 2014/15

Symplectic Geometry

Problem Set 1

1. Let g be a Riemannian metric on $\mathbb{R}^n$, and consider the Lagrangian function $L : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ given by $L(t, x, v) = g_x(v, v)$. Prove that the Euler-Lagrange equations for the corresponding variational problem are the geodesic equations for the metric g!

2. Prove that a linear subspace W of codimension 1 in a symplectic vector space (V, ω) is always coisotropic!

3. **a)** Prove that $\mathrm{Sp}(2, \mathbb{R})$ is diffeomorphic to the open solid torus $S^1 \times \mathbb{R}^2$!

b) Let $\Delta := \{\Psi \in \mathrm{Sp}(2, \mathbb{R}) \mid \det(\mathbb{1} - \Psi) = 0\}$. Prove that $\mathrm{Sp}(2, \mathbb{R}) \setminus \Delta$ consists of two connected components, one diffeomorphic to $\mathbb{R}^3$ and the other diffeomorphic to $S^1 \times \mathbb{R}^2$!

4. Let (V, ω) be a symplectic vector space and consider a coisotropic subspace $W \subset V$. Prove:

a) The quotient space $\overline{W} := W/W^\perp$ carries a natural symplectic form induced from V.

b) If $L \subset V$ is a Lagrangian subspace, then $\overline{L} := ((L \cap W) + W^\perp)/W^\perp$ is a Lagrangian subspace of $\overline{W}$.