

Seminar on the h-cobordism theorem

Preliminary list of talks

This is a mathematical seminar, for which learning mathematical content and learning to present material that has been extracted from the literature through self-study in a form appropriate to the given audience are both equally important goals.

- 1. The h-cobordism theorem and some applications** [3, §9]
- 2. The cobordism category, introduction to Morse functions** [3, §§1 and 2 up to Thm. 2.5]
Present the basic definitions and properties of cobordisms. Also, discuss the classification of 1- and 2-dimensional manifolds up to cobordism. Introduce Morse functions.
- 3. Existence of Morse functions** [3, remaining part of §2]
Prove the existence of Morse functions on any smooth manifold triad, and discuss some consequences.
- 4. Gradient like vector fields** [3, §3 up to and including Cor. 3.8]
Discuss gradient like vector fields, prove the collar and bicollar theorems and deduce that cobordisms can be smoothly glued, and that the Morse number is subadditive.
- 5. CW complexes and cellular homology** [2, chap. 2] and/or [1, chap. 4] (2 talks?)
Give the basic definitions of (singular) homology and discuss examples. Discuss the long exact sequence of a pair and cellular homology.
- 6. Elementary cobordisms** [3, remaining parts of §3]
Define elementary cobordisms and prove the main structure theorem (3.14) for them.
- 7. Rearrangement of cobordisms** [3, §4]
State and prove the rearrangement theorem.
- 8. Cancellation Theorem** [3, §5] (2 talks)
State and prove the first cancellation theorem.

9. The Whitney trick [3, §6] (2 talks)

State and prove the second cancellation theorem.

10. Cancellation of critical points [3, §§7 and 8] (2 talks)

Discuss the results of §§7 and 8 about cancellation of critical points.

References

- [1] G. Bredon, *Topology and Geometry*, Springer GTM 139, 1993
- [2] A. Hatcher, *Algebraic topology*, <http://www.math.cornell.edu/~hatcher/AT/ATpage.html>
- [3] J. Milnor, *Lectures on the h-cobordism theorem*, Princeton University Press, 1965