Universität Hamburg
Janko Latschev
Andreas Gerstenberger

Winter 2012/13

Symplectic Geometry

Problem Set 1

1. Recall that for a linear subspace W of a symplectic vector space (V, ω) we defined the ω-orthogonal complement as

$$W^{\perp_\omega} := \{v \in V \,:\, \omega(w, v) = 0 \text{ for all } w \in W\}.$$

a) Prove that $\dim W + \dim W^{\perp_\omega} = \dim V$!

b) Prove that $(W^{\perp_\omega})^{\perp_\omega} = W$!

c) Prove that W is a symplectic subspace if and only if $(W, \omega|_W)$ is a symplectic vector space if and only if $V = W \oplus W^{\perp_\omega}$!

2. Prove that a linear subspace W of codimension 1 in a symplectic vector space (V, ω) is always coisotropic!

3. Prove that $\mathrm{Sp}(2, \mathbb{R})$ is diffeomorphic to the open solid torus $S^1 \times \mathbb{R}^2$!

4. Give examples of elements of $\mathrm{SL}(4, \mathbb{R})$ which are not elements of $\mathrm{Sp}(4, \mathbb{R})$!

5. Prove that a 2-form ω on a $2n$-dimensional real vector space V is symplectic if and only if

$$\omega^n = \omega \wedge \cdots \wedge \omega \neq 0.$$