Errata. 1° The formula representing divisibility, $m|n$, in §43 should be

$$n = 0 \lor m = \underline{1} \lor \exists x(x < n \land n = m \cdot x).$$

2° Nor is representing relative primeness as easy as it is made to appear there; luckily no representation is needed in the proof of the Lemma.

50 Creative sets are complete

Theorem 49.4 suggests a method for proving a set A productive: show $\bar{K} \leq_m A$. We will see that this method works for every productive A.

50.1 Creative Set Theorem (Myhill, 1955).

(i) If P is productive, then $\bar{K} \leq_1 P$.

(ii) If C is creative, then C is 1-complete, and $C \equiv K$.

Proof of (i). Let p be a 1-1 productive function for P. Take an index e such that

$$\varphi_e(x, y, z) \begin{cases} = z \text{ if } y \in K \text{ and } z = p(x), \\ \uparrow \text{ otherwise.} \end{cases}$$

Put $f(x, y) = s^2_1(e, x, y)$. Then

$$W_{f(x,y)} = \begin{cases} \{p(x)\} \text{ if } y \in K, \\ \emptyset \text{ otherwise.} \end{cases}$$

By the Parametrized Recursion Theorem (32.3), there is a 1-1 computable function n such that

$$W_{n(y)} = W_{f(n(y),y)} = \begin{cases} \{p(n(y))\} \text{ if } y \in K, \\ \emptyset \text{ otherwise.} \end{cases}$$

The composite pn is 1-1 and computable. Moreover,

$$y \in K \Rightarrow W_{n(y)} = \{pn(y)\} \Rightarrow pn(y) \notin P,$$
$$y \notin K \Rightarrow W_{n(y)} = \emptyset \subseteq P \Rightarrow pn(y) \in P. \qquad \boxtimes$$

50.2 Corollary. The statements (i) P is productive, (ii) $\bar{K} \leq_1 P$, and (iii) $\bar{K} \leq_m P$ — are equivalent.

50.3 Corollary. The statements (i) C is creative, (ii) C is 1-complete, and (iii) C is m-complete — are equivalent.