

Homework 4 (due Monday 24 September)

Exercises 18:2 and 18:3 from the lecture notes, which are also Exercises 1.5.18 and 1.5.19 from the Syllabus.

18:2. Prove that $\text{deg}_m(A \oplus B) = \text{deg}_m(A) \vee \text{deg}_m(B)$ (least upper bound), namely

(a) $A \leq_m A \oplus B$ and $B \leq_m A \oplus B$ and

(b) if $A \leq_m C$ and $B \leq_m C$ then $A \oplus B \leq_m C$. [20pts]

18:3. Prove that K_0, K_1 and K are 1-equivalent (i.e., $K_0 \equiv_1 K_1 \equiv_1 K$). [30 pts]

HINT FROM SOARES. Note that the proof of Theorem 1.5.10 (syllabus) automatically shows that $K \leq_1 A$ for $A = K_1, \text{Con}$, or Inf . Use the same method with K_0 in place of K to show that $K_0 \leq_1 K$ and hence $K_0 \leq_1 K \leq_1 K_1$. See also Theorem 2.6.2.