Homework 13 (due Thursday 29 November)

Exercises 2.6.11. and 2.6.12. from the syllabus.

Exercise 2.6.11. If A is c.e. and $A \cap K = \varnothing$, prove that $A \cup K \equiv K$ (note that by Theorem 1.6.4., $\equiv$ is the same as $\equiv_1$, i.e. it is sufficient to prove $A \cup K \leq_1 K$ and $K \leq_1 A \cup K$.) [20 pts]

Exercise 2.5.12. Let $\mathrm{Ind}_x = \{y \mid \varphi_x = \varphi_y\}$.

(a.) Prove that for each x, Ind_x is productive. [20 pts]

Hint. Note that Lemma 2.5.8. proved this for φ_{e_0} where $\varphi_{e_0}(x) \uparrow$ for all x (the nowhere defined function), and that that proof works if $\mathrm{dom}(\varphi_x) \neq \omega$. If $\mathrm{dom}(\varphi_x) = \omega$ then combine Corollary 2.6.7. and Exercise 1.5.21.

(b.) Show that the reduction $\overline{K} \leq_m \mathrm{Ind}_x$ (which you should have used in the proof of (a.)) is not uniform in x, namely that there is no computable function $f(x, y)$ such that for all x we have $y \in \overline{K}$ iff $f(x, y) \in \mathrm{Ind}_x$. [20pts]

Hint. Choose $y_0 \in K$, consider $\lambda x(f(x, y_0))$, and use the Recursion Theorem on f.