Homework 12 (due Monday 26 November)

Prove Example 49.2.3 from the lecture notes.

Example 49.2.3: Let T be an ω-consistent[1] extension of N, and let C be the set of all Gödel numbers of theorems of T, i.e.

$$C := \{n \mid n = \ulcorner \phi \urcorner \text{ for some } \phi \text{ such that } T \vdash \phi\}$$

Prove that C is creative. [30pts]

HINT: Let m be the Gödel number of the formula $\phi(x_1) \equiv \exists y\, \tau(\underline{x}, x_1, y)$, where τ represents Kleene's T-predicate as in 49.2.2. (Kleene's T-predicate is the relation $\mathrm{T}(e, x, y)$ which holds iff y codes the computation of Turing machine with index e, on input x. Cf. Theorem 1.4.3.) Clearly m can be computed from x; and if $W_x \subseteq \overline{C}$, then $Sub(m, m) \in \overline{X} - W_x$.

[1] A theory T in the language of arithmetic is **ω-consistent** if for every $\phi(x)$ the following holds: if for every $n \in \omega$ we have $T \vdash \phi(\underline{n})$, then $T \nvdash \exists x \neg\phi(x)$.