

Homework 11 (due Thursday 22 November)

Exercise on Page 39, Section 47 of the lecture notes.

Exercise: Suppose T is an effectively formalized extension of N . All the notions like Prv refer to provability in T .

- Suppose $\psi(x, y)$ represents “ y codes a proof of the formula with Gödel number $Sub(x, x)$ ”. Precisely:

$$\psi(x, y) \equiv Prv(Sub(x, x), y)$$

- Let $\phi(x)$ be the formula $\neg\exists y \psi(x, y)$ or equivalently $\forall y \neg\psi(x, y)$, i.e., $\phi(x)$ says “there is no y such that y codes a proof of the formula with Gödel number $Sub(x, x)$ ”.
- Let n be the Gödel number of $\phi(x)$.
- Show that if T is consistent, then

$$T \not\vdash \phi(\underline{n})$$

[30 pts]

REMARK: The notions Sub and Prv aren't actually formal symbols in the language of T , but they are just a computable function and a computable relation (in the meta-theory). However, since they are computable, they are **representable** by some formulas. So when we write e.g. “ $\psi(x, y) \equiv Prv(Sub(x, x), y)$ ” we are actually referring to the representations of Prv and Sub . In the notation we suppress the difference between these two notions, in the same way as when we said “+ represents addition”, “= represents equality” and so on.