Homework 8, due Tuesday 22 April, 12:00

1. Show that in the Rieger-Nishimura ladder, $w_i \models g_n(p)$ iff $w_n R w_i$. You can assume that the cases $n = 0, 1, 2$ are true. [6 pts]

2. Show that if $\vdash_{\mathbf{IPC}} g_{n+5}(\varphi)$ then $\vdash_{\mathbf{IPC}} g_i(\varphi)$ for some $i < n + 5$.

   This is a step in a more proof-theoretic proof of Theorem 40 of the lecture notes.

   Use exercise 2 (a) from Homework 5, namely that if $\vdash_{\mathbf{IPC}} (\varphi \to \psi) \to \chi \vee \theta$ then $\vdash_{\mathbf{IPC}} (\varphi \to \psi) \to \chi$ or $\vdash_{\mathbf{IPC}} (\varphi \to \psi) \to \theta$ or $\vdash_{\mathbf{IPC}} (\varphi \to \psi) \to \varphi$.

   Use also that $\vdash_{\mathbf{IPC}} \varphi(p) \leftrightarrow \psi(p)$ iff $\varphi(p)$ and $\psi(p)$ have the same value in the Rieger-Nishimura lattice (see page 56 slides) where the value of implications is calculated as on page 64 slides (and disjunctions in the obvious manner). [6 pts]

3. Assume $\mathfrak{M} \models \neg\neg\varphi$ and $\mathfrak{M} \not\models \varphi$. Assume that $\mathfrak{N} \models \neg\varphi$. Now add nodes w'_n for $n \geq 3$ to the union of $\mathfrak{M}$ and $\mathfrak{N}$ to obtain a new model $\mathfrak{M}^*$. Describe the accessibility relation of the new model.

   With the help of the p-morphism of exercise 1 (b) of Homework 7, construct a p-morphism from $\mathfrak{M}^*$ to the Rieger-Nishimura ladder. Use then exercise 1 (a) of Homework 7 to show that $\mathfrak{M}^* \not\models g_n(\varphi)$ for each n.

   Conclude theorem 40. [6 pts]