Homework 3, due Monday 21 February

1. Show that **KC** (= **IPC** + $\neg\varphi \vee \neg\neg\varphi$) can be axiomatized by its axioms for atomic formulas only (i.e., we get the same logic if we only add the sentences $\neg p \vee \neg\neg p$ for all propositional letters p). [5 pts]

2. Prove directly from Glivenko's theorem (if $\vdash_{\mathbf{CPC}} \varphi$ then $\vdash_{\mathbf{IPC}} \neg\neg\varphi$) that:

   If $\psi_1, \ldots, \psi_k \vdash_{\mathbf{CPC}} \varphi$ then $\neg\neg\psi_1, \ldots, \neg\neg\psi_k \vdash_{\mathbf{IPC}} \neg\neg\varphi$. [3 pts]

3. (a) Show that **LC** characterizes the upwards linear frames ($\forall x, y, z(xRy \wedge xRz \to yRz \vee zRy)$), i.e., show that $\mathfrak{F} \models \mathbf{LC}$ iff R is upwards linear. [2 pts]

   (b) Show that **KC** characterizes the upwards directed frames ($\forall x, y, z(xRy \wedge xRz \to \exists w(yRw \wedge zRw))$). [2 pts]

4.* **Definition:**

   - φ is **negative** iff there is some ψ such that $\vdash_{\mathbf{IPC}} \varphi \leftrightarrow \neg\psi$
   - φ has the **down property** iff for each w which is not an end-point, if for all x with wRx and $w \neq x$ we have $x \models \varphi$, then $w \models \varphi$.

   Show that φ is negative iff it has the down property (we did one direction essentially in class but do it neverhteless). [4pts]

5. Show that, if Γ is a maximal propositional theory that does not prove φ (i.e. $\Gamma \nvdash \varphi$ and, if $\Gamma \subset \Delta$, then $\Delta \vdash \varphi$), then Γ has the DP (disjunction property). [2 pts]