Practice exercises for Monday 12 November

1. Let π be the *Mostowski collapse* from $(N, \in)$ to $(M, \in)$, where N is not transitive and M is transitive. Show that if $S \subseteq N$ is transitive, then $\pi(a) = a$ for every $a \in S$.

2. A formula ϕ is called *arithmetic* if $\phi(x_1, \ldots, x_n) \to x_1, \ldots, x_n \in V_\omega$ for all $x_1, \ldots, x_n$, and all quantifiers appearing in ϕ are of the form $\exists x \in V_\omega$ or $\forall x \in V_\omega$. Show that ϕ is absolute for every transitive model of set theory.

3. The *Twin Prime Conjecture* is the statement that there are infinitely many prime numbers $p \in \omega$ such that $p + 2$ is also prime (currently one of the largest open problems in number theory). Argue why, if the *Twin Prime Conjecture* is independent from ZFC, then this independence cannot be shown by producing a transitive model of $\mathsf{ZFC} + \mathsf{TPC}$ and another transitive model for $\mathsf{ZFC} + \neg\mathsf{TPC}$ (one would need methods using non-transitive models)