Sheet 5

Question 5.1

Considering the short exact sequence of sheaves

$$0 \to \underline{\mathbb{Z}} \xrightarrow{m\cdot} \underline{\mathbb{Z}} \to \underline{\mathbb{Z}/m} \to 0$$

for m prime deduce an expression for $H^i(X, \underline{\mathbb{Z}/m})$ in terms of $H^j(X, \underline{\mathbb{Z}})$.

Question 5.2

Find a topological space X' and a continuous map $f : X' \to X$ such that the map $\mathcal{F} \to \prod \mathcal{F}_x$ identifies with the unit morphism of the adjunction $f^{-1} \dashv f_*$.

Question 5.3

Let X_i be a collection of toploogical spaces and consider the coproduct $X = \amalg_i X_i$ with inclusions $\iota_j : X_j \to X$. For each i let $\mathcal{F}_i$ be a sheaf of abelian groups on X_i.
Compute the cohomology $H^*(X, \oplus_i (\iota_i)_* \mathcal{F}_i)$ in terms of $H^*(X_i, \mathcal{F}_i)$.

Question 5.4

Compute $H^*(\mathbb{R}P^2, A)$ for any abelian group A.

Question 5.5 *

Let X be the Warsaw circle defined as $S \cup L \cup A \subset \mathbb{R}^2$ with

$$\begin{aligned} S &= \{(x-1, 1+\sin\left(\frac{\pi}{x}\right)) \mid x \in (0,1]\} \\ L &= \{(-1, y) \mid y \in [0,2]\} \\ A &= \{(x,y) \mid x^2 + y^2 = 1 \wedge (y \leq 0 \vee x \geq 0))\} \end{aligned}$$

Compute Čech cohomology of X with coefficients in $\underline{\mathbb{Z}}$. Compare with other topological invariants you know.

These questions will be discussed in the exercise class on 16 May 2025.

Questions with an asterisk are more challenging.