Universität Hamburg
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Sheaves in Topology (Master)
Summer Semester 2025

Sheet 3

Question 3.1

Let $0 \to A \to B \to C \to 0$. Show that if A is injective or C is projective the short exact sequence splits.

Question 3.2

Let R be an abelian group and let $X = S^1 = A \cup B$ with $A, B \cong [0,1]$ and $A \cap B \cong [0,1] \amalg [0,1]$. Denote the closed inclusions by j_A, j_B and $j_{A\cap B}$. Define suitable maps such that

$$0 \to \underline{R} \to (j_A)_*\underline{R} \oplus (j_B)_*\underline{R} \to (j_{A\cap B})_*\underline{R} \to 0$$

is a short exact sequence of sheaves.

Show that the image of this sequence under the global section functor is not exact by computing a cokernel.

Question 3.3

Let k be a field, X a topological space and $\iota : \{x\} \to X$ the inclusion of a point. Show that the skyscraper sheaf $\iota_* k$ is Γ-acyclic.

Question 3.4 *

Show that the category of sheaves of abelian groups on $\mathbb{R}$ does not have enough projectives.

These questions will be discussed in the exercise class on 2 May 25.

Questions with an asterisk are more challenging.