

Sheet 12

Question 12.1

Let K be a Klein bottle and $p : K \rightarrow *$ the projection. Compute $H^*(K, p^! \mathbb{Z})$.

Question 12.2

Show a sheaf \mathcal{F} on X is c -soft if for all opens $U \subset X$ we have $H_c^i(U, \mathcal{F}|_U) = 0$ for $i > 0$.

Hint: One of the statements of Proposition 5.17 is helpful.

Question 12.3

Assume $f : X \rightarrow Y$ is a map of cohomological dimension at most n . Show that for any exact sequence of sheaves on X

$$S^0 \rightarrow \cdots \rightarrow S^{n-1} \rightarrow T \rightarrow 0$$

if each S^i is f -soft then so is T .

Question 12.4

Show that for $f : X \rightarrow Y$ a map between locally compact spaces $f_!$ commutes with arbitrary direct sums.

These questions will be discussed in the exercise class on 11 July 2025.

Questions with an asterisk are more challenging.