

## Sheet 1

### Question 1.1

Fix the topological space  $X = \mathbb{R}^n$  and define the presheaf  $B$  of bounded continuous functions with values in  $\mathbb{R}$ . Show it is not a sheaf and compute its sheafification.

### Question 1.2

Let  $x \in X$  be a point in  $\mathbb{R}^n$  (or in an arbitrary topological space such that  $\{x\}$  is closed and not open). Consider the presheaf  $\mathbb{Z}_x$  on  $X$  defined by  $\mathbb{Z}_x(U) = \mathbb{Z}$  if  $x \in U$  and 0 otherwise.

- (a) Is  $\mathbb{Z}_x$  a sheaf?
- (b) Determine all presheaf homomorphisms from  $\mathbb{Z}_x$  to  $\underline{\mathbb{Z}}$  and from  $\underline{\mathbb{Z}}$  to  $\mathbb{Z}_x$ .

### Question 1.3

Let  $j : V \rightarrow X$  be the inclusion of a connected proper open subset into  $\mathbb{R}^n$  (or into any connected topological space).

Let  $\mathbb{Z}'$  be the presheaf sending  $U$  to  $\mathbb{Z}$  if  $U \subset V$  and 0 otherwise. Let  $\mathbb{Z}_!$  be the sheafification of  $\mathbb{Z}'$ .

- (a) Determine all stalks of  $\mathbb{Z}_!$ .
- (b) Compute  $\mathbb{Z}_!(X)$ .
- (c) Determine all sheaf homomorphisms from  $\mathbb{Z}_!$  to  $\mathbb{Z}$  and from  $\mathbb{Z}$  to  $\mathbb{Z}_!$ .

**These questions will be discussed in the exercise class on Tuesday 15/4/25.**

Questions with an asterisk are more challenging.