Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

Julian Holstein
Rational Homotopy Theory (Master)
Winter Semester 2020/21

Sheet 6

Question 6.1

Show that the the following diagram of cdga's has a lift:

$$\begin{array}{ccc} \mathbb{Q}\langle z\rangle & \longrightarrow & A \\ {\scriptstyle\theta}\big\downarrow & & \big\downarrow{\scriptstyle\sim} \\ \mathbb{Q}\langle y, dy\rangle & \longrightarrow & B \end{array}$$

Here $\theta : z \mapsto dy$ as in lectures and $A \to B$ is any acyclic fibration.

Question 6.2

In the model category $\mathsf{cdgA}_{\mathbb{Q}}^{\geq 0}$ compute the homotopy fiber of $\mathbb{Q} \to \mathbb{Q}\langle t\rangle$ for t a generator in some non-negative degree. (It suffices to replace one map by a fibration.)

Question 6.3

In the model category $\mathsf{cdgA}_{\mathbb{Q}}^{\geq 0}$ compute the homotopy cofiber of $\mathbb{Q}\langle t\rangle \to \mathbb{Q}$ defined by $t \mapsto 0$. Here t a generator in some non-negative degree. (It suffices to replace one map by a cofibration.)

Question 6.4

Show that any class of maps defined by satisfying a LLP is closed under retracts.

Question 6.5 (*)

Let I be the diagram category $* \leftarrow * \rightarrow *$ and let $\mathcal{M}$ be a model category such that $\mathcal{M}^I$ has a model structure where weak equivaleneces and fibration are defined objectwise.

Let $A \leftarrow B \rightarrow C$ be an object in $\mathcal{M}^I$ such that B is cofibrant and $B \to A$ and $C \to A$ are two cofibrations.

Show that $A \leftarrow B \rightarrow C$ has the LLP with respect to acyclic fibrations.