Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

Julian Holstein
Rational Homotopy Theory (Master)
Winter Semester 2020/21

Sheet 2

Question 2.1

Show that the cohomology of a cdga is a graded commutative algebra, as claimed in lectures.

Question 2.2

Show that in the cdga $\Omega(n)$ defined in lectures we have $df = \frac{\partial f}{\partial t_i} dt_i$ for $f = f(t_1, \ldots, t_n) \in \Omega(n)^0$.

Question 2.3

Let M be a connected semi-free cdga with $M^1 = 0$. Show that if the differential of M is decomposable then M is minimal.

Question 2.4

Give an example of a connected semi-free cdga with decomposable differential that is not minimal.

Question 2.5

* Find all generators up to degree 4 of the minimal model for $(S^2 \times S^3)\#(S^3 \times S^2)$.

Here the symbol $\#$ denotes the *connected sum*. For any two n-manifolds A and B the connected sum $A\#B$ is defined as $A' \amalg_{S^{n-1}} B'$ where A' and B' are obtained by removing an open n-ball from A and B respectively. The boundary $(n-1)$-spheres are then identified. For example $(S^1 \times S^1)\#(S^1 \times S^1)$ is a surface of genus 2.

These questions will be discussed in the exercise class on 16.11.20.

Questions with an asterisk are more challenging.