Universität Hamburg
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Algebraic Topology (Master)
Summer Semester 2023

Exercise sheet 9

Question 9.1

Recall that for any n, k there is a map $S^n \to S^n$ of degree k. For which values of n and k can this map be chosen fixed point free?

Question 9.2

Calculate $H^m(\mathbb{R}P^n; \mathbb{Z}/2\mathbb{Z})$ and $H^m(\mathbb{R}P^n; \mathbb{Z})$ for all $m \geq 0$ and $n \geq 1$.

Question 9.3

Compute the chomology groups of the Moore space $M(A, n)$ for A a finitely generated abelian group.

Can you construct a CW complex which has cohomology isomorphic to A concentrated in degree n?

Question 9.4

a) Check that $\mathrm{Hom}_{\mathsf{Ab}}(-, G)$ is left exact for any abelian group G,

b) Determine $\mathrm{Ext}(A, B)$ if A is a finitely generated abelian group.

c) For natural numbers n and m give an explicit formula for $\mathrm{Ext}(\mathbb{Z}/n\mathbb{Z}, \mathbb{Z}/m\mathbb{Z})$.

Question 9.5

* Prove Corollary 17.6: Let f be a simplicial homeomorphism of a finite simplicial complex K. Then $\tau(f) = \chi(K^f)$ where K^f is the subspace of fixed points of $|K|$.

These questions will be discussed in the class on 14/6/23. You may hand in your solutions the day before.

Questions with an asterisk are more challenging.