Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

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Algebraic Topology (Master)
Summer Semester 2023

Exercise sheet 4

Question 4.1

Prove the Brouwer fixed-point theorem: Let f be a continuous self map of the closed ball $\mathbb{D}^n$, $n \geq 0$. Prove that f has a fixed point.

Question 4.2

Let $n \geq 0$ be any natural number. Can you find a pair of spaces (X_n, A_n) such that A_n is not the empty set and

$$H_0(X_n, A_n) \cong H_0(X_n \setminus A_n) \cong \mathbb{Z}^n?$$

Question 4.3

Take a closed orientable surface of genus g, Σ_g, and use excision to prove that $H_2(\Sigma_g, F_g \setminus \{x\}) \cong \mathbb{Z}$ for $x \in \Sigma_g$.

Do the same with the Möbius strip, M. Pick a generator $\mu_x \in H_2(M, M \setminus \{x\})$. Assum x is on the meridian of M. What happens with the generator μ_x if x walks along the meridian of the Möbius strip?

Question 4.4

Use the Mayer-Vietoris theorem and suitable covers to compute the homology groups of

a) the torus,

b) the Klein bottle

Question 4.5

* Show that $H^1(X, A)$ is not isomorphic to $H^1(X/A)$ if $X = [0, 1]$ and $A = \{0\} \cup \bigcup_{n>0} \{\frac{1}{n}\}$.

These questions will be discussed in the class on 3/5/23. You may hand in your solutions (in pairs) the day before.

Questions with an asterisk are more challenging.