Universität Hamburg
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Julian Holstein
Algebraic Topology (Master)
Summer Semester 2023

Exercise sheet 2

Question 2.1

Let A_*, B_*, C_* be chain complexes and suppose we have chain maps $f, g : A_* \to B_*$ and $k : B_* \to C_*$. Assume further that h is a chain homotopy from f to g. Show that $k \circ f$ is chain homotopic to $k \circ g$.

Question 2.2

Let C_* be an arbitrary chain complex and let p be a prime. Is it always true that the sequence of chain complexes

$$0 \longrightarrow C_* \xrightarrow{p\cdot} C_* \xrightarrow{\pi} C_*/pC_* \longrightarrow 0$$

is exact? Give a proof or a counterexample.

Question 2.3

a) Let X and Y be topological spaces. Is every chain map $f_* \colon S_*(X) \to S_*(Y)$ induced by a map of topological spaces?

b) Let $p \colon \tilde{X} \to X$ be a covering map. We know that the induced map on fundamental groups is a monomorphism. Is that also true for $H_1(p)$?

Question 2.4

a) Check the claim from lectures that $H_1(S^1 \vee S^1) \cong \mathbb{Z} \oplus \mathbb{Z}$.

b) Let F_g denote the closed orientable surface of genus g. Use the Seifert-van Kampen theorem to determine the fundamental group of F_g and then apply the Hurewicz theorem to calculate $H_1(F_g)$.

c) Do the same for the Klein bottle, K.

d) * Simplify your work by stating and proving a Seifert-van Kampen theorem for H_1.

These questions will be discussed in the class on 19/4/23. You may hand in your solutions (in pairs) the day before.

Questions with an asterisk are more challenging.