Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

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Algebraic Topology (Master)
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# Exercise sheet 11

### Question 11.1

a) Let $\alpha \in H^1(\mathbb{R}P^2; \mathbb{Z}/2\mathbb{Z})$ and $a \in H_1(\mathbb{R}P^2; \mathbb{Z}/2\mathbb{Z})$ be generators. What is $\alpha \cap a$?

b) Let $a$ and $b$ be the 1-cycles corresponding to meridian and longitude of the torus $T^2$. Let $\alpha, \beta$ be the dual classes in cohomology. Compute directly the cup products $\alpha \cup \beta$ and $\beta \cup \alpha$. You may want to use the 2-cycle from Example 6.6.

### Question 11.2

Let $\Sigma_g$ be the surface of genus $g$. Construct a natural quotient map $\Sigma_g \to \vee_g \Sigma_1$ by contracting a subspace homeomorphic to a sphere with $g$ holes down to a point.

Compute the cohomology ring of the right hand side and use it to compute the cohomology ring on the left hand side.

### Question 11.3

Show that for $\partial\colon H^*(A) \to H^{*+1}(X, A)$, $\alpha \in H^*(A)$ and $\beta \in H^*(X)$ the cup product satisfies:

$$\partial(\alpha \cup i^*\beta) = (\partial\alpha) \cup \beta \in H^*(X, A).$$

### Question 11.4

Show that the suspension of a CW complex is a CW complex.

### Question 11.5

Is $\Sigma(S^1 \times S^1)$ homotopy equivalent to $S^2 \vee S^2 \vee S^3$? You may assume that the homotopy classes of pointed maps from $S^2$ into any space, $\pi_2(X)$ form an abelian group under concatenation.

* What about $\Sigma(X \times Y)$ and $\Sigma X \vee \Sigma Y \vee \Sigma(X \wedge Y)$ in general? (Here $\wedge$ denotes the *smash product* $X \times Y / X \vee Y$ of well-pointed spaces.)

**These questions will be discussed in the class on 28/6/23. You may hand in your solutions the day before.**

Questions with an asterisk are more challenging.