Calculus – 3. Series

(turn in: November 7, 2003)

1. (a) Compute real part, imaginary part, and absolute value of the complex numbers

$$(1-7\mathrm{i})(4+3\mathrm{i}), \quad \frac{2+3\mathrm{i}}{1-4\mathrm{i}}, \quad (1-7\mathrm{i})^2, \quad 5\left(\cos\frac{5\pi}{3}+\mathrm{i}\sin\frac{5\pi}{3}\right).$$

   (b) Determine the polar form of the following complex numbers

$$-2\mathrm{i}, \quad 1-\mathrm{i}, \quad -\sqrt{3}-\mathrm{i}.$$

2. Which subsets of the complex plane are described by the following inequalities?

$$\text{a) } |\,z-1\,| \le 3, \quad \text{b) } (z-\mathrm{i})(\overline{z}+\mathrm{i}) \ge 1, \quad \text{c) } z+\overline{z} \ge -1.$$

3. Use de Moivre formula to express

$$\cos 3\alpha \quad \text{and} \quad \sin 4\alpha$$

   in terms of $\cos\alpha$ and $\sin\alpha$.

4. Solve for $z \in \mathbb{C}$.

$$\text{a) } z^6+8\mathrm{i}=0, \quad \text{b) } z^2+\mathrm{i}=0.$$

5. Find the mistake in the following deduction. Let $a, b \in \mathbb{R}$ with $a > b$. Then

$$\begin{aligned}
\sqrt{a-b} &= \sqrt{(-1)(b-a)} = \sqrt{-1}\sqrt{b-a} \\
\sqrt{a-b} &= \sqrt{-1}\sqrt{(-1)(a-b)} = \sqrt{-1}\sqrt{-1}\sqrt{a-b} \qquad \Big|\cdot\frac{1}{\sqrt{a-b}} \\
1 &= \sqrt{-1}\sqrt{-1} = \mathrm{i}^2 \\
1 &= \mathrm{i}^2.
\end{aligned}$$