Matroid theory: exercise sheet 6

1. Let k be a field. Prove that the class of matroids which are representable over k is closed under 2-sums.

2. (a) Let M be a matroid and F a set with 2 elements such that $|E(M) \cap F| = 1$. Prove that $M \cong M \oplus_2 U_{1,F}$.

   (b) Let M be a matroid and let $e_0 \in E(M)$ be an element which is neither a loop nor a coloop. Prove that M has a minor of the form $U_{1,F}$ with $e_0 \in F$ and $|F| = 2$.

   (c) Let M_1 and M_2 be matroids on the sets E_1 and E_2 with $E_1 \cap E_2 = \{e_0\}$, and suppose that e_0 is neither a loop nor a coloop of M_1 or M_2. Prove that $M_1 \oplus_2 M_2$ has an M_1-minor and an M_2-minor.

3. Find all simple 3-connected graphs G with the property that there is no edge e of G such that $G \backslash e$ or G/e is simple and 3-connected.

4* Let G and H be simple 3-connected graphs such that $M(G) \cong M(H)$. Prove that $G \cong H$.